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THESIS

AN ADAPTATION OF A
MARKOV CHAIN MODEL FOR
ANTISUBMARINE WARFARE CARRIER AIRCRAFT

by

George Maurice Lanman

May 1966

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ABSTRACT

It is the purpose of this paper to develop a useful mathematical model of ASW aircraft availability. The increasing emphasis of systems studies dictates the use of accurate and representative models of the ASW systems. At present, many studies are using essentially the same models developed during World War II. This paper is an attempt to make use of advanced theory in a more powerful and flexible model and to make the use of the model practical and verifiable.

The writer adapted the time homogeneous bivariate model as developed by F. C. Collins. This is a discrete time Markov process with a stochastic matrix of transition probabilities wherein the maintenance process is modeled as a pulsed input multiple server queue.

The model was programmed in FORTRAN 63 on the CDC 1604 and then modified to allow for variability in the input parameters. Other modifications include an increase in the size of the model to accommodate a 16-aircraft squadron, the largest ASW squadron at present, and an explicit form solution to the maintenance queueing equations.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS

Symbol	Definition or Meaning
a/c	aircraft
λ	mean repair rate of aircraft
λ_A	mean accident rate
S	the set of all possible outcomes; the probability description space
E	possible outcome(s) or event(s)
$p_{ij}(n, n+1)$	a conditional probability that at time n + 1 the outcome or state is j given that at time n the state is i
$X_1(t)$	the number of a/c flying at time t, which did not fly the previous cycle
$X_2(t)$	the number of a/c in the maintenance queue at time t
A(t)	the number of a/c desired on station at time t
N(t)	the total number of a/c of type considered at time t
T	the time interval from the launch to recovery at the start of the cycle
$Q(t_o)$	the probability distribution over all possible states at initial time t_o
P(t)	the matrix of transition probabilities at time t

Symbol	Definition or Meaning
$p_{(\alpha, i)(\beta, j)}$	the elements of the P matrix; the probability that $X_1 = \beta$ and $X_2 = j$ at the end of a cycle, given that $X_1 = \alpha$ and $X_2 = i$ at the start of the cycle
γ_{fgh}	the probability given f ready a/c, g are launched, and h enter maintenance
p_γ	probability of entering maintenance just before, during, or immediately after launch
p	the probability of equipment failure during flight requiring maintenance when recovered by the carrier
$\Pi_\alpha(m)$	the probability that of α a/c flying m will enter maintenance upon recovery
D	the number of independent identical maintenance repair stations or "spots"
$p_{ij}(t)$	the probability that i - j a/c are repaired in time interval t

1. INTRODUCTION

The threat to freedom of the seas posed by the vast Soviet submarine fleet is perhaps the most thorny problem facing the U. S. Navy today. Two world wars have produced Pyrrhic victories over limited submarine fleets. During the Second World War operations analysis was born into the Navy to aid in the defeat of the German submarine. The classic antisubmarine warfare (ASW) analyses and models developed by Morse [2] and Koopmans [3] are still being used today, over two decades later, in most of the ASW study efforts for the Navy.

These early ASW analyses assumed a given level of search effort available and directly evaluated the probability that an ASW subsystem could detect and/or kill a submarine. This assumption is not only logical to make the problem tractable, but also practical since no immediate changes in ASW force levels could be expected. Moreover, the studies were conducted during the war, not before it started. It is the purpose of this paper to present a probabilistic model to describe the available effort. Such a model can be used to sharpen the estimates of the effectiveness of an ASW subsystem and to study the characteristics of the associated support system.

Naturally, the current study plays an important but limited role in the overall problem of designing an entire ASW system. The difficulties involved in such a specification are legion. First and foremost

is the quantification of the ASW mission in denying the enemy the effective use of his submarines. Currently, the probability of detecting and/or killing submarines is used as the measure of effectiveness of the mission, and it appears that a more encompassing one has not been developed. Second, the specification of an ASW force level to counter a given threat has many inherent subjective elements. These are due to the existing historical bias in predicting the conduct of a future ASW war with an enemy, particularly one who has never before used a large submarine force in its military operations. The reader can imagine why merely defining terms such as "threat" and "effective counter" becomes quite difficult.

Thus, there is a need to investigate the levels of search effort specified. This may require acceptable models to measure the availability of effort, its effectiveness, and determine the logistic support required for any level of available effort. Specifically, the ASW subsystem to be modeled is the carrier-based aircraft, although the model is adaptable to other systems.

The method of investigating the demand for ASW carrier a/c will assume that the desired number of a/c on station is known as an input parameter. The support required to achieve this measure of available effort depends upon maintenance space, manpower, and supply. Generally, we shall consider how an ASW carrier supports this number of a/c on station with the present or proposed number of a/c embarked

on the carrier. The parametric input can be subjected to sensitivity analyses.

The operational commander of the ASW force launches the desired number of a/c on station to screen, search, or actively prosecute a submarine contact. Each a/c is relieved on station. Each such relief requires the launching of another a/c prior to the recovery of the initial a/c. The returning a/c must receive varying degrees of maintenance and requires refueling and rearming. This cycle continues until the mission is completed. Loss of a/c due to accidents, insufficient supply, and lack of repair capability cause deviations in this procedure. Naval operations involve the interaction of many quantities which are random in nature. Not all can be considered in a tractable mathematical model. Some quantities which are important are omitted. One example is the length of each cycle time, which is assumed to be a constant value. Including variables of this nature incurs unnecessary mathematical complication. It is hoped that adequacy of the model can be measured by using fleet data available from the Fleet ASW Data Analysis Program (FADAP).

Collins [5] describes a bivariate Markov model for airborne early warning (AEW) and combat air patrol (CAP) jet a/c operating in an attack carrier force. This model is used to evaluate the probability of maintaining a fixed requirement of a/c on station as a measure of effectiveness of the system. It has subsequently been used in a larger

attack force study for the Navy. The model computes the probabilities of the number of a/c on station and in or awaiting maintenance at any given launch period. The comparable ASW problem differs in the following aspects:

1. Type, range, and speed of a/c;
2. The variable number of a/c required for mission;
3. Attrition due to accidents and supply failures;
4. The greater number of ASW a/c.

It was decided to use the Collins' model with appropriate modification. For immediate reference, the mathematical content of the model will be repeated herein.

In order to incorporate these modifications, it was necessary to spend some time reprogramming on the CDC 1604 digital computer in FORTRAN 63, the CDC version of the IBM FORTRAN IV. The original program was not readily available and was written in an early assembler language. Moreover, the numerical analysis was not sufficiently sharp to handle the larger input values. Also, double precision (two computer words instead of one) arithmetic was required in one subroutine for an accurate explicit solution to the maintenance queueing equations (see Appendix I). This effected a 50% decrease in the computer time required for developing a matrix of transition probabilities.

Following this introduction, section 2 contains a brief description of the operational problems involved and the assumptions made. A brief

description of Markov chains and the mathematical model are presented in section 3. The details for computing the matrix of transition probabilities are given in section 4. General employment of the model follows. The appendices include the solution mentioned on the preceding page, a logical flow diagram of the program, a copy of the program, and some sample results.

C. ASSUMPTIONS

The real-world employment of carrier a/c is cyclic in nature, and the present state of any given a/c (i. e., flying, in or awaiting maintenance) depends largely on what the previous state was. This fact suggests that a Markovian assumption can logically be made for the a/c transition probabilities. In the search phase, a/c may or may not relieve on station; but, in any part of the contact investigation phase, relief on station will be made. To insure full screening and mission coverage, a/c will relieve on station.

The question of resupply during an operation depends primarily on the availability of carrier on-board delivery (COD). This depends on the geographical location and the mission (convoy protection, strike-force protection, hunter-killer operation, etc.). In practice, resupply is not anticipated within a week's period, and around-the-clock operations have continued for two weeks without resupply.

Standard maintenance procedures aboard carriers preclude major maintenance on the flight deck. It will be assumed that sufficient notice is given so that all major 120-hour checks will be completed prior to the operation. This assumption can be modified with an appropriate adjustment in the mean repair rate. The concept of maintenance crews assigned to hangar deck areas ("spots"), as developed by Collins [3], will be used. Each crew will be capable of all types of maintenance

and will operate independently at the identical mean repair rate λ. The number of spots is determined by the average number of such crews available to work continuously around the clock on a watch basis.

The state of each a/c is assumed to be statistically independent of that of others, and the launching and landing transition probabilities will be developed on the basis of independent Bernoulli trials. The parameters can be determined using the maximum likelihood estimators. The range of the number of a/c desired on station at any given cycle will be set by the user. The number to be launched at any time is assumed equally likely within this range. This input parameter is a function of the estimated submarine density (i.e., expected contact rate). The lower limit will be set at the number of a/c desired on station in the search (screening) phase, and the upper limit is set at the maximum practicable number of a/c to be launched during a multiple-contact phase.

Briefly, the assumptions are:

1. a/c will be relieved on station.
2. Any desired length of operation can be set as an input.
3. Major 120-hour checks will be completed prior to the operation.
4. No resupply to the carrier is available.
5. The launch-to-launch cycle for all ASW a/c is four hours.
6. Minor maintenance, refueling, and rearming only can be performed on the flight deck.

7. Each maintenance spot is characterized with an independent exponential repair time with mean repair rate of λ for around-the-clock operations.

8. The number of a/c lost due to attrition is a Poisson random variable for each cycle period with parameter λ_A (a/c accident/flying hours for a/c type).

9. Any a/c lost by accident will not be returned to service due to either (a) physical loss at sea, or (b) insufficient maintenance capability aboard ship and lack of major parts.

10. The number of a/c launched for each cycle is uniformly distributed between the upper and lower limits determined by the user.

3. MODEL DESCRIPTION

3.1 The Theory

A stochastic or random process is a collection of random variables indexed on some set T, $(X(t), t \epsilon T)$. In this case, time is the indexing set, and the Markovian assumption states that the future state of the process depends only on the state at the present time and not on its past history. Due to the cyclic nature of our problem, it is possible to increment time $(T = (0, 1, \ldots))$ using the cycle time from launch to launch as the steps of unit time in a discrete Markov chain. It is assumed that the reader is familiar with the notion of a random variable as a function defined on a sample description space (S) on which the family of events or outcomes (E) of a probability function can be defined [4].

A discrete time Markov chain is described by a sequence of discrete valued random variables and is determined when the one-step transition probabilities of the state variables are specified, i.e., a conditional transition probability of a transition at time n for each pair of i, j = 0, 1, . . . , m (m being the number of states in the process) must be given.

$$p_{ij}(n, n+1) = P[X(n+1) = j \mid X(n) = i]$$

If the transition probability functions depend only on the time difference, we have time homogeneity

$$p_{ij}(n+1\,,\ 1) = p_{ij}(0,\ 1) = p_{ij}\ .$$

The initial state of the system must be given either as a specific state or randomly as a probability distribution function over the possible states.

The p_{ij} (transition probabilities) are arranged in matrix form and satisfy:

1. $p_{ij} \geq 0$ for $i, j = 0, 1, \ldots, m$;
2. $\sum_{j=0}^{m} p_{ij} = 1$, i.e., the rows of the transition matrix sum to 1 for all i for the states within the description space [4].

3.2 The Model

In order to establish the finite set of states (E) for the model, we shall consider two random variables defined as follows:

$X_1(t)$ = The number of a/c flying at time t not having flown in the previous launch-to-launch interval.

$X_2(t)$ = The number of a/c in or awaiting maintenance at time t.

Now, we will consider the vector $X(t) = [X_1(t), X_2(t)]$ as a pair of random variables and thereby have a bivariate stochastic process with the possible states ranging from (0, 0) to (A, N).

$0 \leq X_1(t) \leq A$ = No. of a/c desired on station, and

$0 \leq X_2(t) \leq N$ = No. of a/c of given type aboard carrier.

We will define an operating cycle as an interval unit of time. Process observations of X(t) will be made at successive unit interval launch times. To develop the p_{ij} elements, consider a given time t for launching until A aircraft are flying or until the supply of ready a/c is depleted. Those a/c failing the launch enter the maintenance state at this idealized point in time t (the total launching time required is much less than the total cycle time). At some time T, less than the launch-to-launch unit time interval, the a/c which were relieved on station return and land at the idealized point in time t + T. Some of these a/c will require maintenance and enter the maintenance queue. Those requiring only refueling and preflight inspection will enter a ready status to be tested for the next launch.

During the unit time interval, maintenance will be performed on those a/c in the not-ready status, and a certain number of aircraft will be repaired according to assumption 7.

In summary, we start the system in some initial state (such as (0, 0) with no a/c flying or in maintenance) or start with a probability distribution $Q(t_o)$ over the states, E, at time t_o. We launch, recover, and repair a/c in the unit interval and repeat the process over each succeeding unit time interval until the end of the operating period. Knowing the transition probabilities within the unit time interval, we can develop the elements of the transition matrix, P, or $\{p_{(\alpha, i), (\beta, j)}\}$. These are the probabilities of going from the state of α a/c flying and i a/c in maintenance to β a/c flying and j a/c in maintenance over the unit time interval.

It was assumed in section 2 that A, the number of a/c to be launched, and N, the total number of a/c on board, are random variables, whereas they have been treated as constants so far in the development. To be analytically correct in including this feature, one should develop the appropriate quadrivariate process. Such a development leads to too large a state space and the author chose to include these effects by using a Monte Carlo simulation technique. That is, at the beginning of each cycle, a random mechanism is used to determine the values on A and N.

The probability of losing an a/c or changing the desired number to be launched is determined from the specified distributions at the beginning

of each unit interval, and the resulting P matrix containing the $p_{(\alpha, i), (\beta, j)}$ is then recomputed. The probability distribution $Q(t)$ over the states at any time t may be determined by the appropriate number of successive iterations of the Q vector times the P matrix, i. e.,

$$Q(t) = P[X_1(t) = \beta, X_2(t) = j] = Q(t-1) \times P .$$

The probability of maintaining α a/c on station over any given period of operation may be obtained at any unit time t (i. e., the beginning of the next cycle) by summing out the appropriate maintenance state probabilities. Thus, $P(\alpha$ a/c are flying at time $t) =$

$$Pr(X_1(t) = \alpha) = \sum_{i=o}^{N} Pr(X_1(t) = \alpha, X_2(t) = i) .$$

A mathematical comment appears to be in order. In the case of fixed A and N, the states of the Markov chain are positive recurrent; and steady-state probabilities can be found for the entire state space. In the case of decreasing N due to a/c attrition, this is not true; and (0, 0) becomes an absorbing state as time (t) goes to infinity. This latter consideration is not a realistic one for the operational period envisioned. Therefore, it is mathematically more feasible to use the former chain in conjunction with the Monte Carlo technique.

4. DEVELOPMENT OF THE TRANSITION MATRIX

Perhaps the simplest way to view this development is to note the various transition probabilities incorporated in one-unit time cycle defined as follows:

(1) γ_{fgh} = the launching transition probabilities at time t. This is the probability of taking f ready a/c, launching g successfully, and sending h into maintenance. Each a/c to be launched is considered a Bernoulli trial with probability of failure of p_γ, which is estimable and subject to sensitivity analysis. The values of γ_{fgh} are:

a. 0 if $g > A$, since only A a/c are desired;

b. 0 if $g + h > f$; it is impossible to launch and send into maintenance more a/c than are available;

c. 0 if $g < A$, $g + h < f$; launching continues until A a/c are flying or until all f are used up;

d. $\binom{f}{g} (1 - p_\gamma)^g (p_\gamma)^{f-g}$ if $g < A$, $g + h = f$, standard binomial when all a/c in the ready state are used up but the A a/c are not launched;

e. $\binom{g+h-1}{h} (1 - p)^g (p)^h$ if $g = A$, $g + h > f$, standard negative binomial for g successes in $g + h - 1$ trials.

(2) $\Pi_\alpha(m)$ = the landing transition probabilities which occur at time $t+T$. We must consider the probability that if there are a/c flying at time t then m a/c will enter maintenance at recovery time $t+T$.

$\Pi_\alpha(m)$ will equal a standard binomial where p = the probability of equipment failure in flight:

$$\Pi_\alpha(m) = \binom{\alpha}{m}(1-p)^{\alpha-m}(p)^m, \quad m = 0, 1, \ldots, \alpha .$$

(3) $p_{ij}(\tau)$ = the maintenance transition probabilities, i.e., the probability of repairing (i - j) a/c in time τ. Two maintenance periods occur: the first starting at time t and ending at time t + T, the second starting at time t + T and ending at the end of the cycle, (t + 1). Under assumption 7, the pulsed input, multiple exponential server queue is developed with D maintenance "spots" or servers each with identical, independent service rates, λ. For each server, then, the probability of remaining occupied (given the server is busy) in time $\tau = e^{-\lambda\tau}$. The probability of becoming free (i.e., repairing an a/c) = $1 - e^{-\lambda\tau}$. The resulting queueing equations are:

A. $dP_{i,n}(t)/dt = -n\lambda P_{i,n}(t) + (n+1)\lambda P_{i,n+1}(t)$ for $0 \le n < D$;

B. $dP_{i,n}(t)/dt = -D\lambda P_{i,n}(t) + D\lambda P_{i,n+1}(t)$ for $n \ge D$.

Three ranges of i (initial queue state), j (final queue state), and D become significant:

a. When $j \le i \le D$, then not all spots are busy since there are fewer a/c in maintenance than spots. Each spot works independently; therefore, the solution to A is the binomial:

$$p_{ij}(t) = \binom{i}{j}(1 - e^{-\lambda t})^{(i-j)} e^{-\lambda t j} .$$

b. When $D \leq j \leq i$, then all spots are occupied throughout the total service time, and the closed form solution to B is the Poisson:

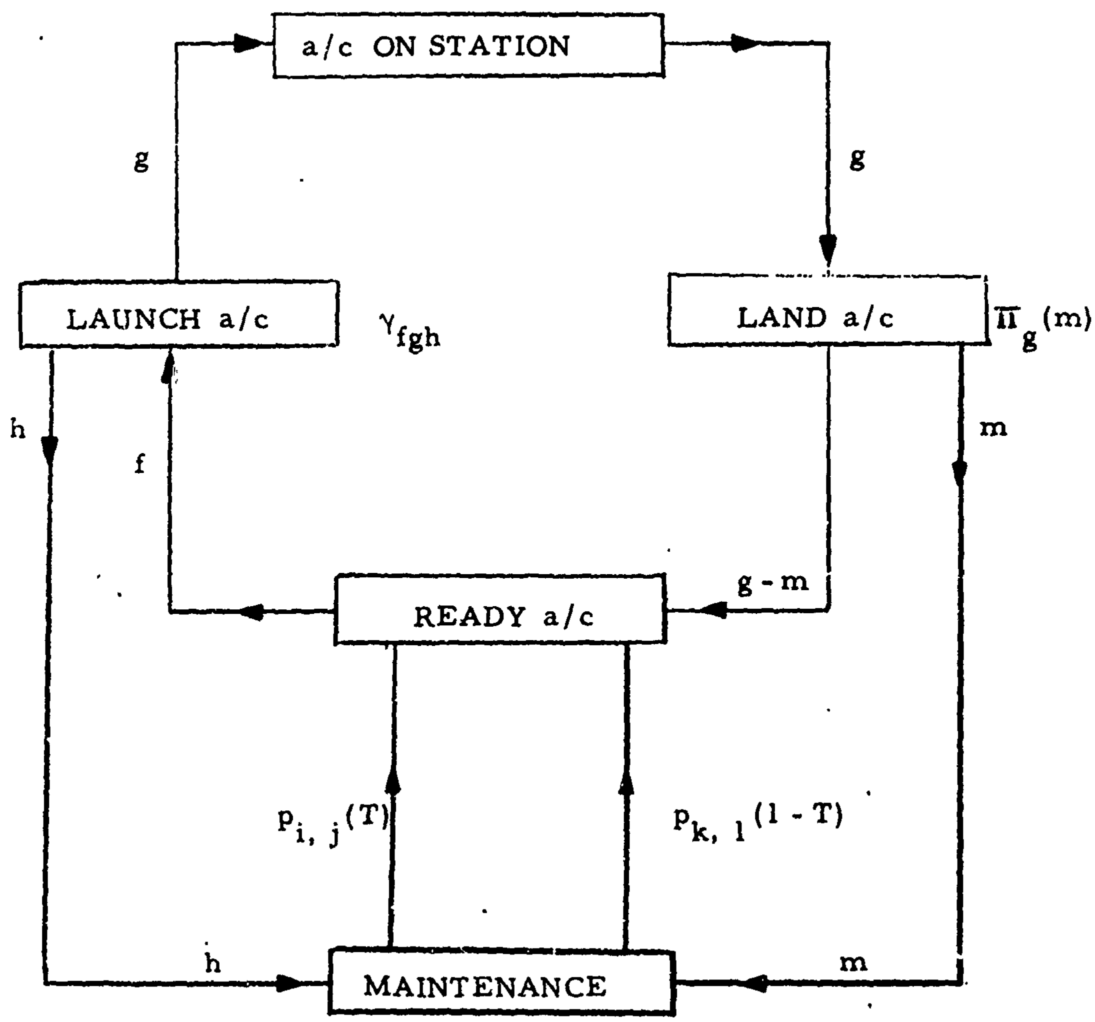
$$p_{ij}(t) = \frac{(D\lambda t)^{(i-j)} e^{-D\lambda t}}{(i-j)!} .$$

c. When $j < D < i$, then all spots are busy at the beginning of the service period, and some spots become idle during the service period. The explicit form solution of equation A is found using moment generating function transformation:

$$p_{ij}(t) = \sum_{n=j}^{D-1} \binom{n}{j} \binom{D}{n} \left\{ \left(\frac{D}{D-n}\right)^{(i-D)} e^{-\lambda t n} - e^{-\lambda t} \sum_{k=o} \left(\frac{\lambda D t}{k!}\right)^{k} \left(\frac{D}{D-n}\right)^{i-D-k} \right\} .$$

(The derivation of this solution is discussed in Appendix I.)

The figure on the following page will show the relationships of these transition probabilities within the unit time interval.



TRANSITION PROBABILITIES WITHIN THE UNIT CYCLE

FIGURE 1

In order to develop each transition probability over the total unit time interval, we must consider all events taking place within the interval. Thus, to obtain the probability of going from α a/c flying and i a/c in maintenance to β a/c flying and j a/c in maintenance, we start at the state (α, i) at time t. At this time, a/c are launched and some l a/c failing the launch enter maintenance. These i + l in maintenance are then serviced until time t + T when some k a/c are still in the maintenance state. At time t + T, of the α a/c previously flying, some m enter maintenance and $(\alpha - m)$ enter the ready pool. Maintenance is continued on the (k + m) a/c for the remainder of the cycle (1 - T), until the end of the unit time interval when j a/c remain in the maintenance state. In functional form:

$$p_{(\alpha, i), (\beta, j)} = \sum_{l=o}^{N-\alpha-i} \sum_{k=o}^{i+l} \sum_{m=o}^{\alpha} \gamma_{N-\alpha-i, \beta, l} \cdot p_{i+l, k}(T) \cdot \Pi_{\alpha}(m) \cdot p_{k+m, j}(1 - T) .$$

5. SUMMARY

Representative values for the mean repair rate and the landing and launching failure rates produced results in agreement with the sensitivity analysis by Collins on these parameters in [5]. For failure probabilities less than .5, and mean repair rate less than 12 hours, the effect of reducing the available maintenance time to 80% of the cycle time was negligible. Optimal loading and cycling policies can be determined for known values of these rates.

The model affords the following checks: (1) the rows of each P matrix are summed as they are computed by the program; and (2) the probability distribution vector (QJ) is summed over the states. Each summation was within 10^{-8} of one in the computer model.

The user may substitute any available distribution over the interval of a/c desired on station. In order to keep A fixed, enter the desired value as both upper and lower limit (A = ALOLIM = LUPLIM). For fixed N, use a very small value for ALAM (such as 10^{-8}). Subroutine KRAN is a uniform generator, using the half open interval (lower limit + 1, upper limit + 2) and a starting number as inputs. KRAN outputs an integer in this interval. Subroutine DRAW was used to provide some intuitive grasp of the results. DRAW was used in binary card form and is not essential to the main program. (The indicated associated statements must be removed, however.)

The results of reasonable arbitrary parameter values, based on the author's experience, have shown that most of the probabilities concentrate over a few states. Moreover, computation time increases rapidly as a function of N (no. of a/c), see Figure 2. This would indicate that a simple approximation to the model could be developed. One method presently being investigated to reduce computation time is to shrink the probability state space to include only those significant states and, thus, reduce the size of the transition matrix. Alternatively, the eigenvector, eigenvalue representation of the P matrix, might be used.

Originally, it was hoped to utilize the data from the Fleet ASW Data Analysis Program (FADAP) to attempt a verification of the model with its real-world counterpart. The only method available at present for obtaining the necessary data is by direct observation or a program of data collection, as suggested by Collins [5].

Many fruitful areas of investigation exist:

(1) Attrition has been simply modeled by the Poisson method. The two components of attrition, accidents and supply shortage, can be more accurately modeled and used to develop logistic schedules for maintenance and supply. One simple technique is to assume each component is independent and Poisson, and estimate a supply failure rate for AOCP attrition from past data. With these assumptions, the total attrition is Poisson, with the parameter equal to the sum of the accident and supply failure rates.

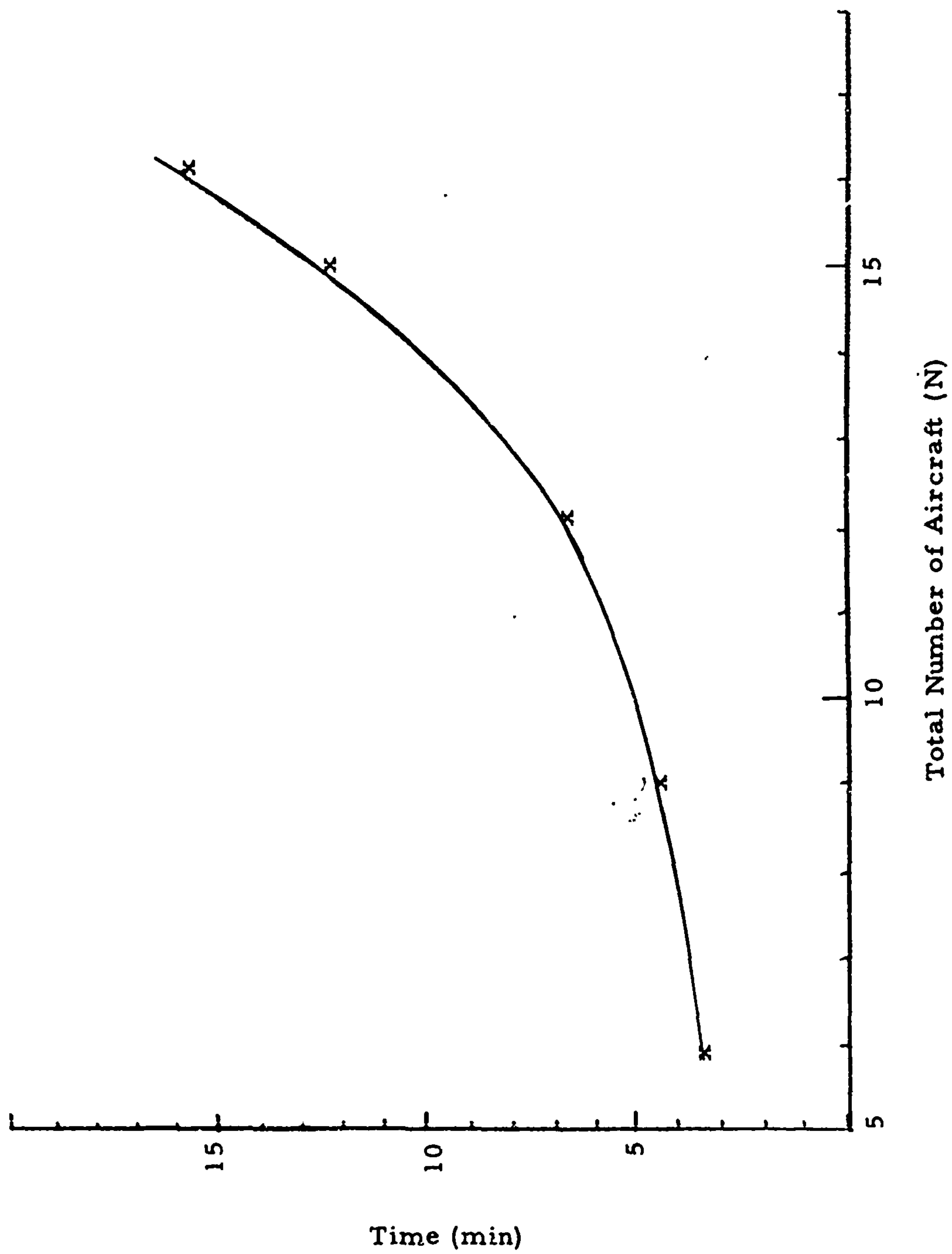
(2) The model can be modified to make the number of maintenance spots available for any cycle a variable function of time, $D(t)$.

(3) An investigation of the Markovian assumption validity as the cycle times become smaller and smaller.

(4) Development of a continuous time model.

(5) Modification of the model to simulate resupply by COD.

(6) A study of the distribution of submarine contacts to determine the validity of the uniform a/c demand assumption.



PROGRAM ASSEMBLY AND COMPUTATION TIME
FOR ONE TRANSITION MATRIX (P) AS A FUNCTION
OF THE TOTAL NUMBER OF AIRCRAFT (N)

FIGURE 2

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APPENDIX I

EXPLICIT SOLUTIONS OF THE MAINTENANCE QUEUEING EQUATIONS

The queueing equations for the pulsed input queue are essentially the pure death process given in [1] and [4] as problems and developed by Collins in [5]. The equations are:

$$\text{A.} \quad \frac{dP_{i,n}(t)}{dt} = -n\lambda P_{i,n}(t) + (n+1)\lambda P_{i,n+1}(t) \quad \text{for } 0 < n < D$$

$$\text{B.} \quad \frac{dP_{i,n}(t)}{dt} = -n\lambda P_{i,n}(t) + D\lambda P_{i,n+1}(t) \quad \text{for } n \geq D$$

where $P_{ij}(0) = \Delta_{ij}$ and $P_{ij}(t) = 0$ for $i < j$, since no input (arrivals) occur during the service time.

Equation B is solved directly in closed form:

$$P_{i,n}(t) = \frac{(D\lambda t)^{(n-i)} e^{-\lambda Dt}}{(n-i)!} .$$

Now transforming the first equation (A) using the moment generating function (MGF),

$$G(s,t) = \sum_{n=0}^{D-1} s^n P_n(t) ,$$

as outlined in [4] (Chapter 7), and its partial derivatives:

(1) $$\frac{dG}{dt} = \sum_{n=0}^{D-1} s^n P_n'(t)$$

(2) $$\frac{dG}{ds} = \sum_{n=0}^{D-1} n s^{n-1} P_n(t)$$

Where $P_n(t)$ denotes the conditional probability $P_{i,n}(t)$, by substituting (A) into (1), properly identifying the first summation with (2), and changing the second summation index to $r = n + 1$, we get:

$$\frac{dG}{dt} = -\lambda s \frac{dG}{ds} + \lambda \sum_{r=0}^{D} r s^{r-1} P_r(t) \,, \qquad \text{or}$$

(3) $$\frac{dG}{dt} = -\lambda (s-1) \frac{dG}{ds} + \lambda D s^{D-1} P_D(t) \,,$$

since

$$\sum_{r=0}^{D} r s^{r-1} P_r(t) = \frac{dG}{ds} + D s^{D-1} P_D(t) \,.$$

Next, replace the partial differential equation (3) with a system of ordinary differential equations using the Lagrangian auxiliary equations:

$$\frac{dt}{1} = \frac{ds}{\lambda (s-1)} = -\frac{dz}{\lambda D s^{D-1} P_D(t)} \,.$$

The solution to the first equation (using the first two differentials) is:

$$\lambda t = \ln (s-1) + C'$$

and hence

$$s = C_1 e^{\lambda t} + 1$$

or

$$G_1 = e^{-\lambda t}(s - 1) .$$

The second equation is: (using first and third differentials)

$$dz = -\lambda D (C_1 e^{\lambda t} + 1)^{D-1} P_D(t)\, dt .$$

Using the solution to (B) where $m = i - D$ to replace $P_D(t)$ and integrating, term wise, the binomial expansion of $(C_1 e^{\lambda t} + 1)^{D-1}$:

$$z = \frac{(\lambda D)^{m+1}}{m!} \sum_{j=0}^{D-1} \binom{D-1}{j} C_1^{\,j} \int t^m e^{-\lambda (D-j) t}\, dt$$

where the integral is evaluated as:

$$- \sum_{k=0}^{m} \frac{t^k e^{-\lambda (D-j) t}}{(\lambda (D-j))^{m-k+1}} \frac{m!}{k!} + C_2 .$$

Thus,

$$C_2 = z + e^{-\lambda Dt} \sum_{j=0}^{D-1} \binom{D}{j} (s-1)^j \sum_{k=0}^{m} \frac{(\lambda Dt)^k}{k!} \left(\frac{D}{D-j}\right)^{m-k}$$

and the general solution is $\phi(C_1, C_2)$, where ϕ is an arbitrary function

and

$$C_1 = u(s, t, z)$$

and

$$C_2 = v(s, t, z) .$$

To get our particular solution, use the boundary conditions for $G(s, t)$:

(1) for $s = 1$,

$$G(1, t) = \sum_{n=0}^{D-1} P_n(t)$$

$$= \Pr [\text{no. in maintenance at } t \text{ is} < D \mid i \text{ at } t = 0]$$

$$G(1, t) = 1 - \sum_{n=0}^{i-D} \frac{e^{-\lambda Dt} (\lambda Dt)^n}{n!} = 1 - \psi_1(t)$$

where

$$u(1, t, z) = C_1 = 0$$

$$v(1, t, z) = C_2 = z + e^{-\lambda Dt} \sum_{k=0}^{m} \frac{(\lambda Dt)^k}{k!}$$

so

$$C_2 = z + \psi_1(t)$$

(2) for $t = 0$,

$$G(s, 0) = \sum_{n=0}^{D-1} s^n P_n(0) = 0 , \quad \text{since } i \geq n > D$$

where

$$u(s, 0, z) = (s - 1)$$

$$v(s, 0, z) = C_2 = z + \sum_{j=0}^{D-1} \binom{D}{j} (s - 1)^j \left(\frac{D}{D - j}\right)^m .$$

Thus,

$$G(s, 0) = z + \sum_{j=0}^{D-1} \binom{D}{j} C^j \left(\frac{D}{D - j}\right)^m - C_2 .$$

Substituting the general value for C_2 above:

$$G(s, t) = \phi(u, v) = \sum_{j=0}^{D-1} \binom{D}{j} (s-1)^j e^{-\lambda t j} \left(\frac{D}{D-j}\right)^m$$

$$- \sum_{j=0}^{D-1} \binom{D}{j} (s-1)^j \sum_{k=0}^{m} \frac{(\lambda D t)^k}{k!} e^{-\lambda D t} \left(\frac{D}{D-j}\right)^{m-k} .$$

Rearranging terms,

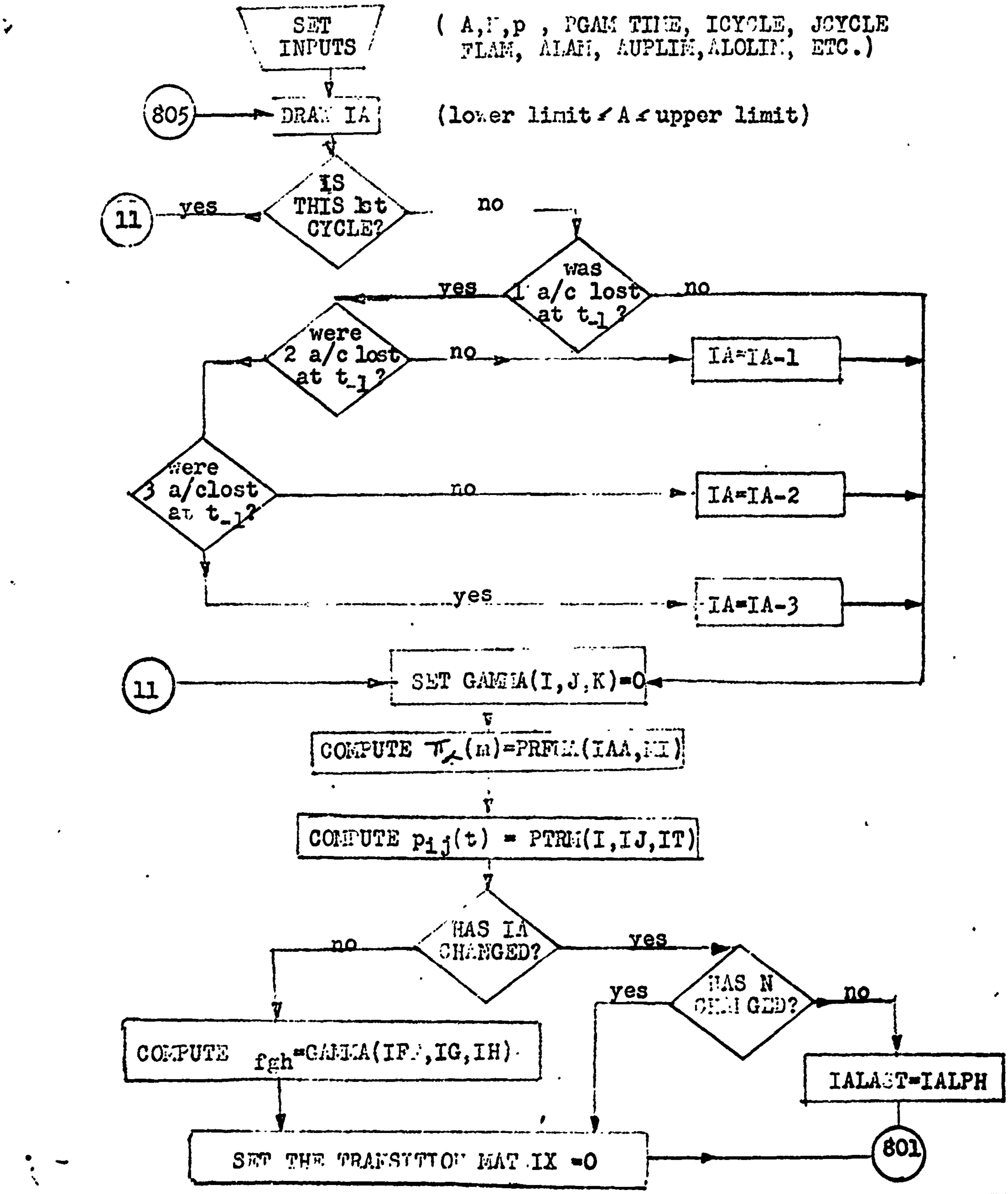
$$G(s, t) = \sum_{n=0}^{D-1} s^n \sum_{j=n}^{D-1} \binom{j}{n} \binom{D}{j} (-1)^j \left[\left(\frac{D}{D-j}\right)^m e^{-\lambda t j} \right.$$

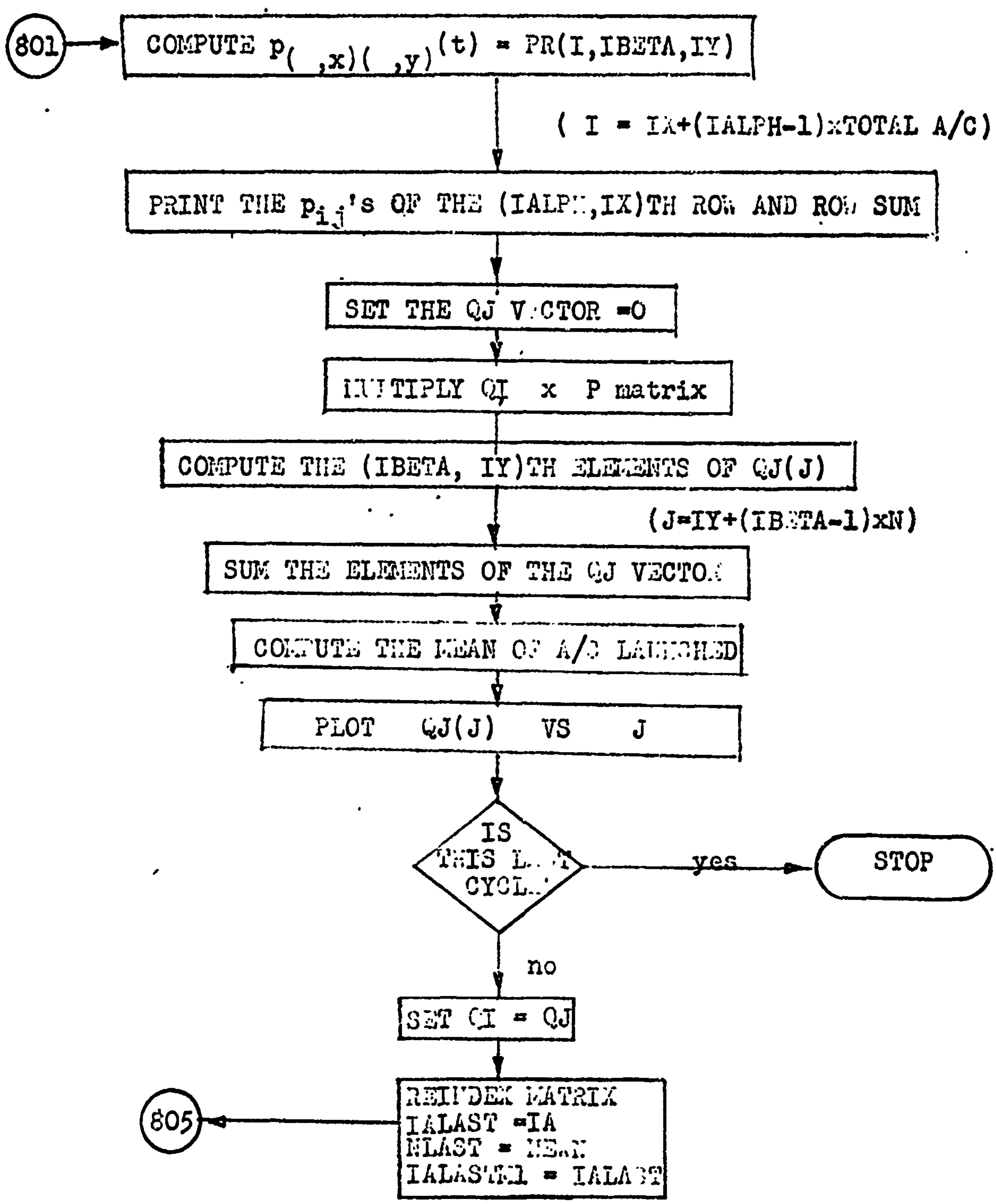
$$\left. - e^{-\lambda D t} \sum_{k=0}^{m} \frac{(\lambda D t)^k}{k!} \left(\frac{D}{D-j}\right)^{m-k} \right]$$

where $P_n(t)$ = the coefficient of s^n .

APPENDIX II

THE LOGICAL FLOW DIAGRAM OF THE COMPUTER PROGRAM




801
COMPUTE p(,x)(,y)(t) = PR(I,IBETA,IY)
(I = IX+(IALPH-1)xTOTAL A/C)
PRINT THE pij's OF THE (IALPH,IX)TH ROW AND ROW SUM
SET THE QJ VECTOR =0
MULTIPLY QI x P matrix
COMPUTE THE (IBETA, IY)TH ELEMENTS OF QJ(J)
(J=IY+(IBETA-1)xN)
SUM THE ELEMENTS OF THE QJ VECTOR
COMPUTE THE MEAN OF A/C LAUNCHED
PLOT QJ(J) VS J
IS THIS LAST CYCLE
yes
STOP
no
SET QI = QJ
REINDEX MATRIX
IALAST =IA
MLAST = MEAN
IALASTM1 = IALAST
805

APPENDIX III

THE COMPUTER PROGRAM

```
-COOP,,LANMAN,0/49/S/15/2S/E/45=54,15 ,30000,5.
-BINARY,56.
(RELOCOM.
-FTN,E.
      PROGRAM MARKOV
C  THIS PROGRAM IS A NONSTATIONARY BIVARIATE MARKOV CHAIN MODEL OF ASW A/C
C  OPERATIONS.  THE RANDOM VARIABLES ARE THE NUMBER OF A/C FLYING AT THE
C  BEGINNING OF ANY GIVEN LAUNCH CYCLE.  THE MAXIMUM NO. OF A/C ALLOWED IN
C  THE MODEL IS 16(NA).  THE RANGE OF A/C TO BE LAUNCHED AT ANY GIVEN
C  INTERVAL IS 0 TO 6 A/C.  THE FOLLOWING INPUTS ARE REQUIRED.
C          ID= THE NO. OF INDEPENDENT MAINTENANCE SPOTS
C          NA= TOTAL NO. OF A/C TYPE ON BOARD
C          TIME=TIME FROM LAUNCH TO RECOVERY/LAUNCH TO LAUNCH CYCLE TIME(HRS)
C          FLAM=MEAN REPAIR TIME PER SPOT/LAUNCH TO LAUNCH CYCLE TIME   (HRS)
C          PGAM= PROBABILITY OF A/C FAILING LAUNCH(M.L.EST. FROM PAST DATA)
C          P= PROBABILITY OF A/C FAILURE DURING FLIGHT REQUIRING MAINTENANCE
C               AT LANDING (M.L. ESTIMATOR FROM PAST DATA)
C          QI= THE PROBABILITY DISTRIBUTION VECTOR OVER ALL POSSIBLE STATES
C            (7 X 17 = 119) SUCH THAT THE SUM OF ALL QI(I) = 1.   THIS
C            IS ESTIMATED BY THE USER AND INPUTTED BY USING  A DATA STATEMENT
C          ICYCLF = NO. CYCLES DESIRED FOR OPERATION
C          JCYCLE = LAUNCH TO LAUNCH TIME(HRS)(TOT. TIME=ICYCLE X JCYCLE)
C          ALAM = ACCIDENT RATE FOR TYPE A/C (ACCIDENT/HOURS)
C          ALOLIM = DESIRED LOWER LIMIT ON A
C          AUPLIM = DESIRED UPPER LIMIT ON A
      COMMON FLAM,TIMF
      TYPE DOUBLE FLAM
      COMMON PTRM,GAMMA,PR,PRFMA,ID
      DIMENSION BC(17),A(17),FBC(17)
      DIMENSION PTRM(17,17,2),GAMMA(17,7,17),PRFMA(7,7)
      DIMENSION PR(119,7,17),QI(119),QJ(119)
      DIMENSION          FJPLOT(119),JT(12)
C     ENTER DATA CARDS HERE
      DATA((QI(I),I=1,119)=.2,16(.05),102(.0))
      NA=16
      ALAM=.01
      ID = 8
       FLAM=3.0
      PGAM=P=.4
      IYY = 13421773
      TIME = .125
      ICYCLE=20
      JCYCLE=4
      ALOLIM=4.
      AUPLIM=6.
C      END OF DATA CARDS
      AL=ALOLIM+1.  $ AU=AUPLIM +2.
      UNITT=1.
      N=NA+1
      IAMAX=7
      IALAST=0
      D=FLOATF(ID)
      NLAST=NEWN=N
      KT=1
  809 IA=KRAN(AL,AU,IYY)
      IF(KT.EQ.1) 113,115
```

```
  115 T1=-LOGF(.000000001 + RANF(-1))*2.30258/ALAM
      IF(T1-TFLC)130,131,132
  130 T2=-LOGF(.000000001 + RANF(-1))*2.30258/ALAM
      IF(T1+T2-TFLC) 230,231,131
  230 T3=-LOGF(.000000001 + RANF(-1))*2.30258/ALAM
      IF(T1+T2+T3-TFLC) 331,331,231
  331 NEWN=NLAST-3  $ GO TO113
  231 NEWN=NLAST-2  $ GO TO113
  132 NEWN=NLAST $ GO TO113
  131 NEWN=NLAST-1
  113 PRINT 8882,IA,NEWN
      IF(NEWN-IA) 15,13,13
   15 IA=NEWN
   13 IF(IALAST) 11,12,11
   12 CONTINUE
C  FROM THIS NEXT STATEMENT TO NO. 483 IS CONCERNED ONLY WITH THE GRAPH
      DO 482 I=1,12
  482 JT(I)=8H
      JT(1)=8HE(A/C) =
      JT(3)=8HSPOTS =
      JT(5)=8H    T =
      JT(7)=8HJ VS QJ
      JT(8)=8HVECTOR
      JT(9)=8H    N =
      JT(11)=8H    A =
      DO 483 I=1,119
      FI=I
  483 FJPLOT(I)=FI
      IALAST=IA
      DO 1235 I=1,17
      DO 1235 J=1,IAMAX
      DO 1235 K=1,17
 1235 GAMMA(I,J,K)=0.0
C  AT THIS PT THE LANDING TRANSITION PROBABILITIES ARE COMPUTED
      DO 300 IAA=1,IAMAX
      DO 301 MI=1,IAMAX
      IF(IAA-MI)31,32,33
   31 PRFMA(IAA,MI)=0.
      GO TO 301
   32 MM1=MI-1
      PRFMA(IAA,MI)=P**MM1
      GO TO 301
   33 IAM1=IAA-1
      MM1=MI-1
      BC(1)=1.0
      PROD=FLOATF(IAA-MI)
      DO 50 IP=2,MI
      AIP=FLOATF(IP-1)
      PROD = PROD +1.0
   50 BC(IP)=PROD*BC(IP-1)/AIP
      IGO=IAA-MI
      PRFMA(IAA,MI)= (BC(MI)*(1.0-P)**(IGO))*P**MM1
  301 CONTINUE
  300 CONTINUE
C  AT THIS PT THE MAINTENANCE TRANSITION PROBABILITIES ARE COMPUTED
      DO 100 IT=1,2
```

```
      IF(IT-1)25,25,26
   25 TAU = TIME
      GO TO 28
   26 TAU = UNITT-TIME
   28 DO101 I=1,N
      DO 102 IJ=1,N
      IF (I-IJ) 14,199,17
  199 IF(I-ID) 19,19,1999
 1999 PTRM(I,IJ,IT)=EXPF(-FLAM*TAU*D)
      GO TO 102
   14 PTRM(I,IJ,IT)=0.
      GO TO 102
   19 FJM1=FLOATF(IJ-1)
      PTRM(I,IJ,IT)=EXPF(-FLAM*TAU*FJM1)
      GO TO 102
   17 IF(I-ID-1) 1,1,2
    1 BC(1)=1.0
      PROD=FLOATF(I-IJ)
      DO 10 IP =2,IJ
      AIP=FLOATF(IP-1)
      PROD = PROD + 1.0
   10 BC(IP) =PROD*BC(IP-1)/AIP
      ELT=EXPF(-FLAM*TAU)
      PTRM (I,IJ,IT)=BC(IJ)*(1.-ELT)**(I-IJ)*ELT**(IJ-1)
      GO TO 102
    2 IF(IJ-1-ID) 22,24,24
   22 CONTINUE
      CALL PID(I,IJ,IT)
      GO TO 102
   24 D=FLOATF(ID)
      ELDT=EXPF(-D*FLAM*TAU)
      FACT = 1.0
      A(1)=1.0
      MM = I-IJ
      DO 20 M=2,MM
      FACT=FACT+1.0
   20 A(M)=A(M-1)*FACT
  201 PTRM(I,IJ,IT)=(D*FLAM*TAU)**(I-IJ)*ELDT/A(I-IJ)
  102 CONTINUE
  101 CONTINUE
  100 CONTINUE
      GO TO 120
   11 CONTINUE
      IF(IA-IALAST) 120,121,120
  121 IF(NEWN-NLAST)111,117,111
  117 IALPH=IALASTM1 $ GO TO 801
  120 CONTINUE
C  AT THIS POINT THE LAUNCHING TRANSITION PROBABILITIES ARE COMPUTED
      DO 204 IFF=1,N
       IGM = XMINOF (IA,IFF)
      DO 203 IG=1,IGM
      IGM1=IG-1
      DO 202 IH=1,N
      IHM1=IH-1
      BPROD=((1.-PGAM)**IGM1)*(PGAM**IHM1)
   86 IF(IG-IA) 91,87,84
```

```
   91 IF(IG+IHM1-IFF) 84,82,84
   87 IF(IG+IHM1-IFF)85,85,84
   84 GAMMA(IFF,IG,IH)=0.
      GO TO 202
   82 BC(1)=1.0
      PROD=FLOATF(IFF-IG)
      DO 30 IP=2,IG
      AIP=FLOATF(IP-1)
      PROD = PROD + 1.0
   30 BC(IP)=PROD * BC(IP-1)/AIP
      IHM1=IH-1
      TEMP= PGAM**IHM1
      TEMP1=(1.-PGAM)**IGM1
       BPROD = TEMP*TEMP1
      GAMMA(IFF,IG,IH)=BC(IG)*BPROD
      GO TO 202
   85 FBC(1)=1.0
      PROD=FLOATF(IGM1-1)
      DO 40 IP=2,IH
      AIP = FLOATF(IP-1)
      PROD = PROD +1.0
   40 FBC(IP)=PROD*FBC(IP-1)/AIP
      GAMMA(IFF,IG,IH)=FBC(IH)*BPROD
  202 CONTINUE
  203 CONTINUE
  204 CONTINUE
C       REMOVE CARDS FROM HERE TO NO 999 IF PRINT OUT NOT DESIRED
      PRINT 9,(((I,IJ,IT,PTRM(I,IJ,IT),IT=1,2),IJ=1,N),I=1,N)
    9 FORMAT (1H1/(2(6H PTRM(I2,1H,I2,1H,I2,3H) = E14.5)))
      PRINT 99,(((IFF,IG,IH,GAMMA(IFF,IG,IH),IFF=1,N),IG=1,IA),IH=1,N)
   99 FORMAT(1H1/(2(7H GAMMA(I2,1H,I2,1H,I2,3H) = E14.5)))
      PRINT 999,((IAA,MI,PRFMA(IAA,MI),IAA=1,IAMAX),MI=1,IAMAX)
  999 FORMAT(1H1/(2(7H PRFMA(I2,1H,I2,3H) = E14.5)))
C  NOW THE TRANSITION MATRIX MUST BE ZEROED
  111 CONTINUE
      DO 899 J=1,119
      DO  899 K=1,7
      DO 899 L=1,17
  899 PR(J,K,L)=0.0
C  START COMPUTING THE ELEMENTS OF EACH ROW, I=IX+ (ALPHA - 1) X TOTAL A/C
      DO 1000 IALPH=1, IALAST
  801 CONTINUE
      DO 1100 IX=1,NLAST
C COMPUTE THE P ELEMENTS OF THE IAPH,IX ROW AND SUM THE ROW
      TSUM=0.
      I=IX+(IALPH-1)*N
      DO 800 IBETA=1,IA
      RSUM=0.0
      DO 900 IY=1,NEWN
      PR(I,IBETA,IY)=0.
      ILIM=NEWN-IALPH-IX+2
      PSUM=0.0
      SUM=0.0
      SUML=0.0
      DO 500 IL=1,ILIM
      KLIM=IX+IL-1
```

```
      IXPIL = IX +IL - 1
      SUMM=0.
      DO 600 MI=1,IALPH
      SUMK=0.
      DO 700 IK=1,KLIM
      IKPMI = IK +MI-1
      IF(IXPIL-NEWN) 701,701,700
  701 IF(IKPMI-NEWN) 702,702,700
  702 GAMH=GAMMA(ILIM,IBETA,IL)
      PTRMH1 = PTRM(IXPIL,IK,1)
      PRFMAH = PRFMA(IALPH,MI)
      PTRMH2 = PTRM(IKPMI,IY,2)
      SUM = GAMH * PTRMH1 * PRFMAH * PTRMH2
      SUMK=SUMK+SUM
      PSUM=PSUM+SUM
  700 CONTINUE
      SUMM = SUMM + SUMK
  600 CONTINUE
      SUML = SUML +SUMM
      PSUM2 =SUML
  500 CONTINUE
      RSUM=RSUM+PSUM
      PR(I,IBETA,IY)=PSUM
  900 CONTINUE
      TSUM=TSUM+RSUM
  800 CONTINUE
      PRINT 888 ,    TSUM,IALPH,IX
  888 FORMAT (                  7H TSUM =  E15.8,2I5)
 1100 CONTINUE
 1000 CONTINUE
C     REMOVE  CARD FROM HERE TO 889 IF P MATRIX PRINT OUT NOT DESIRED
      DO 889 J=1,17
      DO 889 K=1,7
      DO889 L=1,17
      I=J+(K-1)*N
  889 PRINT 890,(PR(I,LP,L),LP=1,IAMAX),K,J,L
  890 FORMAT(7E14.5,2HJ=I2,5HK=1,A,2HL=I2)
      DO 898 I=1,119
  898 QJ(I)=0.0
C NOW  MULTIPLY  QI AND P TO GET QJ
  805 PRINT 807,KT,IALAST,IA
  807 FORMAT(1H1,13HQ VECTOR CASE I3/// I5,I5)
      DO 802 IBETA=1,7
      DO 902 IY=1,17
CAT THIS POINT CALCULATE THE (IBETA,IY)TH ELEMENT OF THE QJ VECTOR
      J=IY+(IBETA-1)*N
      QP1=0.
      QP=0.
      DO 2001 IALPH=1,7
      DO 2201 IX=1,17
      I=IX+(IALPH-1)*N
      QP1=QI(I)*PR(I,IBETA,IY)
      QP=QP+QP1
 2201 CONTINUE
 2001 CONTINUE
      QJ(J)=QP
```

```
      PRINT 8882,IBETA,IY,J,QP
 8882 FORMAT(2I4,4H QJ(I3,3H )= E14.8)
  902 CONTINUE
  802 CONTINUE
C CHECK THE SUM OF THE Q VECTOR
      QSUM=0.
      DO 808 J=1,119
  808 QSUM=QJ(J)+QSUM
      PRINT 8883,QSUM
 8883 FORMAT(6H QSUM=  E15.9)
      DO 333 I = 18,119
      K = (I-1)/17
        FK=FLOATF(K)
      FMEAN= FK*QJ(I)+FMEAN
  333 CONTINUE
      TFLC=FMEAN*FLOATF(JCYCLE)
      PRINT 335,FMEAN
  335 FORMAT(  17HMEAN A/C FLYING = F10.4)
C    STATEMENTS FROM THIS POINT TO THE CALL DRAW STATEMENT REFER TO  GRAPH
      JT(2)=ICODE(FMEAN)
      JT(4)=ICODE(D)
      FKT=FLOATF(KT)
      JT(6)=ICODE(FKT)
      FN=FLOATF(NEWN-1)
      JT(10)=ICODE(FN)
      FIAA=FLOATF(IA-1)
      JT(12)=ICODE(FIAA)
      CALL DRAW(119,FJPLOT,QJ,0,0,4H    ,JT,0,0,0,0,0,0,8,8,0,LAST  )
      FMEAN = 0.
CNEXT WE MUST MULTIPLY QJ AND P TO GET QK AND SO ON...(QK+...N)
      KT=KT+1
      IF(KT-ICYCLE) 803,803,806
  803 DO 804 I=1,119
  804 QI(I)=QJ(I)
      IALASTM1=IALAST
      IALAST=IA
      NLAST=NEWN
      GO TO  809
  806 STOP 06
      END
      SUBROUTINE PID(I,J,IT)
      COMMON FLAM,TIME
      COMMON PTRM,GAMMA,PR,PRFMA,ID
      TYPE DOUBLE BC,BDC,PROD ,DID3,DID4,DID5,DEXP
      TYPE DOUBLE DAN,DID1,DID2,SUM,DN,ANM1,FAC,COF,PSUM,PTR,FLAM,TAU,D
      DIMENSION PTRM(17,17,2),BC(11),BDC(11)
      DIMENSION GAMMA(17,7,17), PRFMA(7,7), PR(119,7,17)
      D=FLOATF(ID)
      IDP1=ID+1
      IF(IT-1)25,25,26
   25 TAU = TIME
      GO TO 28
   26 TAU= 1.-TIME
   28 CONTINUE
      IMDP1=I-ID
      PTR=0.0
```

```
      PSUM=0.
      DO 200 NJ = J, ID
C   DEVELOP N TAKEN J AT A TIME AND D TAKEN N AT A TIME
      BC(1)=1.0
      PROD=FLOATF(NJ-J)
      DO 10 IP=2,J
      AIP=FLOATF(IP-1)
      PROD = PROD+1.0
   10 BC(IP)=PROD* BC(IP-1)/AIP
      BDC(1)=1.0
      PROD=FLOATF(IDP1-NJ)
      DO 20 IQ=2,NJ
      AIQ=FLOATF(IQ-1)
      PROD=PROD+1.0
   20 BDC(IQ)=PROD*BDC(IQ-1)/AIQ
      COF=BC(J)*BDC(NJ)*(-1)**(NJ-J)
      ANM1=FLOATF(NJ-1)
       DAN=D/(D-ANM1)
      DID4=DEXP(-FLAM*TAU*ANM1)
      DID1=(DAN**(I-IDP1))*DID4
      SUM=0.
      DN=0.
      DO 201 K=1,IMDP1
      FAC=1.
      KM1=K-1
      PROD=0.
      DO 11 IK=1,KM1
      PROD=PROD+1.
   11 FAC=FAC*PROD
      IMIDK=I-ID-K
      SUM=((FLAM*D*TAU)**KM1)*DAN**IMIDK /  FAC
  201 DN=DN+SUM
      DID3=DEXP(-FLAM*D*TAU)
      DID2=DN*DID3
      DID5=DID1-DID2
      PSUM=COF*DID5
  200 PTR =PTR +PSUM
  103 CONTINUE
      PTRM(I,J,IT)=PTR
  102 CONTINUE
  101 CONTINUE
      END
      FUNCTION KRAN(A,B,IY)
C
C     THIS ROUTINE RETURNS AN UNIFORMLY DISTRIBUTED RANDOM INTEGER
C
C     THIS ROUTINE RETURNS A INTEGER RANDOM NUMBER .GE. TO A
C     .LT. B
C     A = BOTTOM LIMIT (INCLUDED) FOR THE RANDOM NUMBER
C     B = TOP LIMIT (NOT INCLUDED) FOR THE RANDOM NUMBER
C     SET IY ONLY ONCE IN MAIN PROGRAM FOR EACH SET OF RANDOM NUMBERS
C     SOME GOOD STARTING VALUES FOR IY FOLLOW
C     13421773
C     33554433
C     8426219
C     42758321
```

```
C      56237485
C      62104023
C      ANY OF THESE MAY BE USED
C
C      THIS ROUTINE MAY BE USED IN FORTRAN 60 OR 63
C
       IY = 3125 * IY
       IY = IY - (IY/67108864) * 67108864
       FY = IY
       KRAN = FY/67108864. * (B-A) + A
       RETURN
       END
          FINIS
-EXECUTER.
```

Q VECTOR CASE 1

5 5

1	1	QJ(1)	=2.64762775E-02
1	2	QJ(2)	=3.02020831E-02
1	3	QJ(3)	=1.72436823E-02
1	4	QJ(4)	=6.68152510E-03
1	5	QJ(5)	=2.02525670E-03
1	6	QJ(6)	=5.29756823E-04
1	7	QJ(7)	=1.30002033E-04
1	8	QJ(8)	=[illegible].22530060E-05
1	9	QJ(9)	=8.58857000E-06
1	10	QJ(10)	=2.86285667E-06
1	11	QJ(11)	=8.34999861E-07
1	12	QJ(12)	=2.08749965E-07
1	13	QJ(13)	=4.34895761E-08
1	14	QJ(14)	=7.24826268E-09
1	15	QJ(15)	=9.06032836E-10
1	16	QJ(16)	=7.55027363E-11
1	17	QJ(17)	=3.14594735E-12
2	1	QJ(18)	=3.02519433E-02
2	2	QJ(19)	=3.07381552E-02
2	3	QJ(20)	=1.54385405E-02
2	4	QJ(21)	=5.18865107E-03
2	5	QJ(22)	=1.34374437E-03
2	6	QJ(23)	=2.96164302E-04
2	7	QJ(24)	=6.07227021E-05
2	8	QJ(25)	=1.26203649E-05
2	9	QJ(26)	=2.86289503E-06
2	10	QJ(27)	=8.35011051E-07
2	11	QJ(28)	=2.08752763E-07
2	12	QJ(29)	=4.34901589E-08
[illegible]	13	QJ(30)	=7.24835981E-09
[illegible]	[illegible]	QJ(31)	=9.06044977E-10
2	15	QJ(32)	=7.55037481E-11
2	16	QJ(33)	=3.14598950E-12
2	17	QJ(34)	= 0
3	1	QJ(35)	=3.40989043E-02
3	2	QJ(36)	=3.07597531F-02
3	3	QJ(37)	=1.35265107E-02
3	4	QJ(38)	=3.91531781E-03
3	5	QJ(39)	=8.57092801E-04
3	6	QJ(40)	=1.56625190E-04
3	7	QJ(41)	=2.62271034E-05
3	8	QJ(42)	=4.44105168E-06
3	9	QJ(43)	=8.35125955E-07
3	10	QJ(44)	=2.08781489E-07
3	11	QJ(45)	=4.34961435E-08
3	12	QJ(46)	=7.24935725E-09
3	13	QJ(47)	=9.06169656E-10
3	14	QJ(48)	=7.55141380E-11
3	15	QJ(49)	=3.14542242E-12
3	16	QJ(50)	= 0
3	17	QJ(51)	= 0
4	1	QJ(52)	=3.79764295E-02
4	2	QJ(53)	=3.03224061E-02
4	3	QJ(54)	=1.16230913E-02
4	4	QJ(55)	=2.87808733E-03
4	5	QJ(56)	=5.26888829E-04
4	6	QJ(57)	=7.84720454E-05
4	7	QJ(58)	=1.04516559E-05
4	8	QJ(59)	=1.39145026E-06
4	9	QJ(60)	=2.08960624E-07
4	10	QJ(61)	=4.35334634E-08
4	11	QJ(62)	=7.25557723E-09
4	12	QJ(63)	=9.06947154E-10
4	13	QJ(64)	=7.55789295E-11
4	14	QJ(65)	=3.14912206E-12
4	15	QJ(66)	= 0
4	16	QJ(67)	= 0
4	17	QJ(68)	= 0
5	1	QJ(69)	=4.91642833E-01
5	2	QJ(70)	=1.43718127E-01
5	3	QJ(71)	=2.69531853E-02
5	4	QJ(72)	=3.78709962E-03
5	5	QJ(73)	=4.23732653E-04
5	6	QJ(74)	=3.99381550E-05
5	7	QJ(75)	=3.41914062E-06
5	8	QJ(76)	=2.98561135E-07
5	9	QJ(77)	=3.11088104E-08
5	10	QJ(78)	=4.95114755E-09
5	11	QJ(79)	=5.92108046E-10
5	12	QJ(80)	=4.72886874E-11
5	13	QJ(81)	=1.89139810E-12
5	14	QJ(82)	= 0
5	15	QJ(83)	= 0
5	16	QJ(84)	= 0
5	17	QJ(85)	= 0
6	1	QJ(86)	= 0
6	2	QJ(87)	= 0
6	3	QJ(88)	= 0
6	4	QJ(89)	= 0
6	5	QJ(90)	= 0
6	6	QJ(91)	= 0
6	7	QJ(92)	= 0
6	8	QJ(93)	= 0
6	9	QJ(94)	= 0
6	10	QJ(95)	= 0
6	11	QJ(96)	= 0
6	12	QJ(97)	= 0
6	13	QJ(98)	= 0
6	14	QJ(99)	= 0
6	15	QJ(100)	= 0
6	16	QJ(101)	= 0
6	17	QJ(102)	= 0

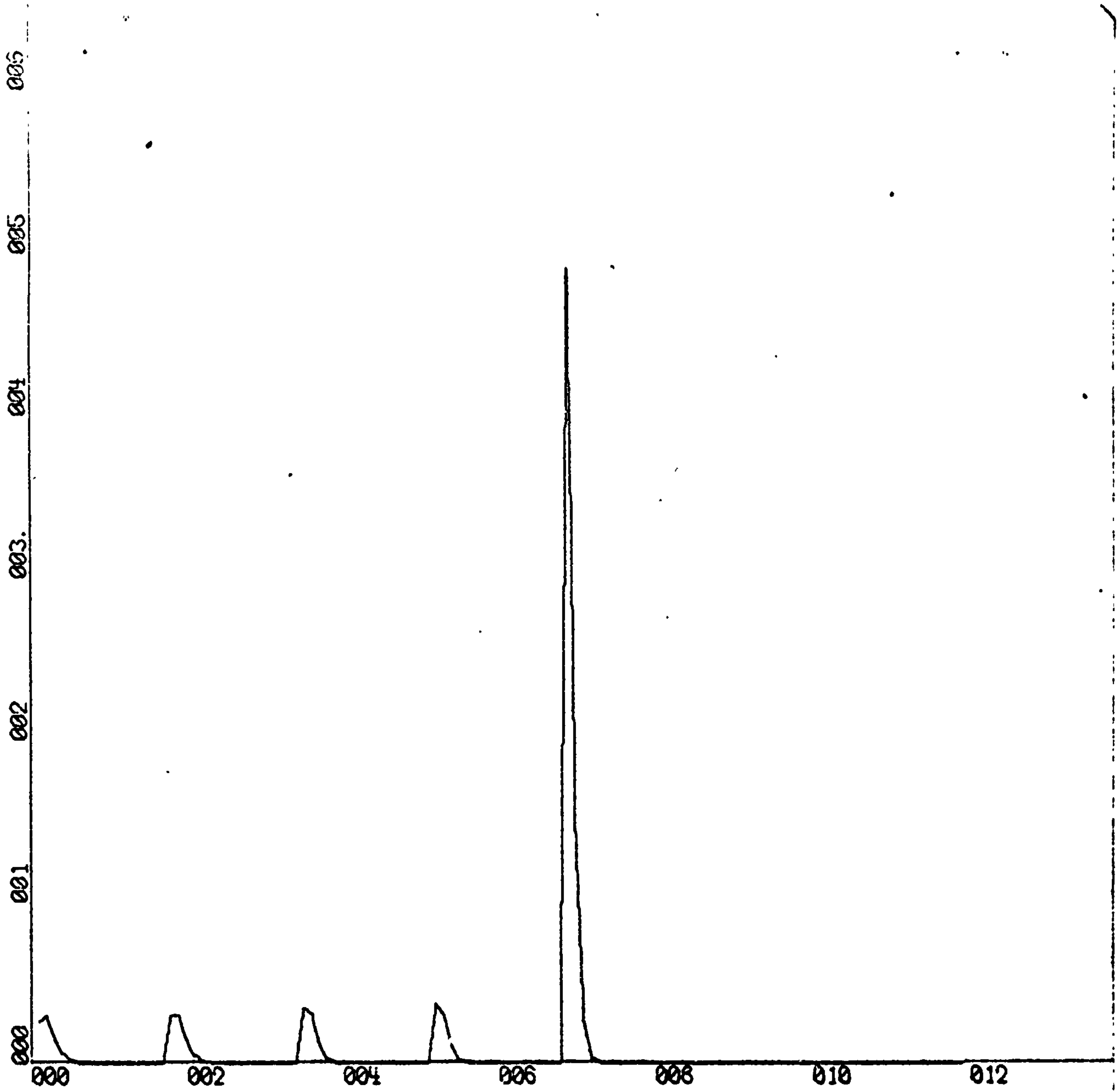
```
  7    1 QJ(103 )=                 0
  7    2 QJ(104 )=                 0
  7    3 QJ(105 )=                 0
  7    4 QJ(106 )=                 0
  7    5 QJ(107 )=                 0
  7    6 QJ(108 )=                 0
  7    7 QJ(109 )=                 0
  7    8 QJ(110 )=                 0
  7    9 QJ(111 )=                 0
  7   10 QJ(112 )=                 0
  7   11 QJ(113 )=                 0
  7   12 QJ(114 )=                 0
  7   13 QJ(115 )=                 0
  7   14 QJ(116 )=                 0
  7   15 QJ(117 )=                 0
  7   16 QJ(118 )=                 0
  7   17 QJ(119 )=                 0
QSUM=1.000000000E 00
EAN A/C FLYING =3.1666E 00

GRAPH TITLED
    E(A/C) =3.17E+00SPOTS = 8.00E+00    T = 1.00E+00
    J VS QJ VECTOR      N = 1.60E+01    A = 4.00E+00
HAS BEEN PLOTTED.
```

APPENDIX IV

SAMPLE RESULTS

The following pages present the values of the elements of the probability distribution vector (Q J) and its graphical plot for five consecutive iterations, i.e., $Q \times P^n$ for n = 1, 2, ..., 5. The inputs are those shown on the first page of Appendix III between statement No. 30 and No. 31. The printouts of the transition matrices and their computational elements are omitted. The plot was made using the DRAW subroutine in the U. S. Naval Postgraduate School computer facility library. Each vector printout contains the values of all 119 states possible (7 x 17) and is headed by the past value of A + 1 and the next value of A + 1. The two indices preceding each element represent $\beta + 1$ and j + 1, in the notation of section 3. For example, in the first row on the next page, the "1 1" indicates that the probability of being in state (0, 0) after one iteration is $\doteq$ .026, where the value of A is 4 over the first iteration. Each graph is labeled with the expected value of a/c flying, the number of maintenance spots available, the vector number (T), total number of a/c available (N), and the desired number of a/c on station (A). The "E" notation indicates the power of 10 to multiply by. This sample run demonstrates the loss in total a/c and variable a/c on station.



X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E-01 UNITS/INCH.

E(A/C) =3.17E+00SPOTS = 8.00E+00   T = 1.00E
J VS QJ VECTOR   N = 1.60E+01   A = 4.0

Q VECTOR CASE 2

5 5

1	1	QJ(1)=	9.12184624E-06	4	1	QJ(52)=	7.23035242E-03
1	2	QJ(2)=	7.94491028E-06	4	2	QJ(53)=	4.45693238E-03
1	3	QJ(3)=	3.41380533E-06	4	3	QJ(54)=	1.28966676E-03
1	4	QJ(4)=	9.83913729E-07	4	4	QJ(55)=	2.35500532E-04
1	5	QJ(5)=	2.20530359E-07	4	5	QJ(56)=	3.11156232E-05
1	6	QJ(6)=	4.28567319E-08	4	6	QJ(57)=	3.30021596E-06
1	7	QJ(7)=	7.97994342E-09	4	7	QJ(58)=	3.15123146E-07
1	8	QJ(8)=	1.56348438E-09	4	8	QJ(59)=	3.13674353E-08
1	9	QJ(9)=	3.46529220E-10	4	9	QJ(60)=	3.81044171E-09
1	10	QJ(10)=	1.01221242E-10	4	10	QJ(61)=	6.98663591E-10
1	11	QJ(11)=	2.58173003E-11	4	11	QJ(62)=	1.02568738E-10
1	12	QJ(12)=	5.63385201E-12	4	12	QJ(63)=	1.13092255E-11
1	13	QJ(13)=	1.02295616E-12	4	13	QJ(64)=	8.32962386E-13
1	14	QJ(14)=	1.48426307E-13	4	14	QJ(65)=	3.07550593E-14
1	15	QJ(15)=	1.61419113E-14	4	15	QJ(66)=	0
1	16	QJ(16)=	1.17030587E-15	4	16	QJ(67)=	0
1	17	QJ(17)=	4.24536876E-17	4	17	QJ(68)=	0
2	1	QJ(18)=	1.69105396E-04	5	1	QJ(69)=	7.70076632E-01
2	2	QJ(19)=	1.31058642E-04	5	2	QJ(70)=	1.88159529E-01
2	3	QJ(20)=	4.93425071E-05	5	3	QJ(71)=	2.32480412E-02
2	4	QJ(21)=	1.22416819E-05	5	4	QJ(72)=	1.93788734E-03
2	5	QJ(22)=	2.31695297E-06	5	5	QJ(73)=	1.22762068E-04
2	6	QJ(23)=	3.73462197E-07	5	6	QJ(74)=	6.39420794E-06
2	7	QJ(24)=	5.70661929E-08	5	7	QJ(75)=	3.02738808E-07
2	8	QJ(25)=	9.22154646E-09	5	8	QJ(76)=	1.56312395E-08
2	9	QJ(26)=	1.72702939E-09	5	9	QJ(77)=	1.13519205E-09
2	10	QJ(27)=	4.41481881E-10	5	10	QJ(78)=	1.50627061E-10
2	11	QJ(28)=	9.65869676E-11	5	11	QJ(79)=	1.51200491E-11
2	12	QJ(29)=	1.75883018E-11	5	12	QJ(80)=	1.02093434E-12
2	13	QJ(30)=	2.56023495E-12	5	13	QJ(81)=	3.47880865E-14
2	14	QJ(31)=	2.79434001E-13	5	14	QJ(82)=	0
2	15	QJ(32)=	2.03393035E-14	5	15	QJ(83)=	0
2	16	QJ(33)=	7.41007441E-16	5	16	QJ(84)=	0
2	17	QJ(34)=	0	5	17	QJ(85)=	0
3	1	QJ(35)=	1.42525319E-03	6	1	QJ(86)=	0
3	2	QJ(36)=	9.83757697E-04	6	2	QJ(87)=	0
3	3	QJ(37)=	3.24331350E-04	6	3	QJ(88)=	0
3	4	QJ(38)=	6.90234010E-05	6	4	QJ(89)=	0
3	5	QJ(39)=	1.09403657E-05	6	5	QJ(90)=	0
3	6	QJ(40)=	1.44032446E-06	6	6	QJ(91)=	0
3	7	QJ(41)=	1.76353392E-07	6	7	QJ(92)=	0
3	8	QJ(42)=	2.28131763E-08	6	8	QJ(93)=	0
3	9	QJ(43)=	3.51078290E-09	6	9	QJ(94)=	0
3	10	QJ(44)=	7.70559333E-10	6	10	QJ(95)=	0
3	11	QJ(45)=	1.40851171E-10	6	11	QJ(96)=	0
3	12	QJ(46)=	2.05928494E-11	6	12	QJ(97)=	0
3	13	QJ(47)=	2.25869291E-12	6	13	QJ(98)=	0
3	14	QJ(48)=	1.65302266E-13	6	14	QJ(99)=	0
3	15	QJ(49)=	6.05799194E-15	6	15	QJ(100)=	0
3	16	QJ(50)=	0	6	16	QJ(101)=	0
3	17	QJ(51)=	0	6	17	QJ(102)=	0

```
  7    1 QJ(103 )=                0
  7    2 QJ(104 )=                0
  7    3 QJ(105 )=                0
  7    4 QJ(106 )=                0
  7    5 QJ(107 )=                0
  7    6 QJ(108 )=                0
  7    7 QJ(109 )=                0
  7    8 QJ(110 )=                0
  7    9 QJ(111 )=                0
  7   10 QJ(112 )=                0
  7   11 QJ(113 )=                0
  7   12 QJ(114 )=                0
  7   13 QJ(115 )=                0
  7   14 QJ(116 )=                0
  7   15 QJ(117 )=                0
  7   16 QJ(118 )=                0
  7   17 QJ(119 )=                0
QSUM=1.000000000E 00
EAN A/C FLYING =3.9799E 00

GRAPH TITLED
    E(A/C) =3.98E+00SPOTS = 8.00E+00    T = 2.00E+00
    J VS QJ VECTOR       N = 1.60E+01    A = 4.00E+00
HAS BEEN PLOTTED.
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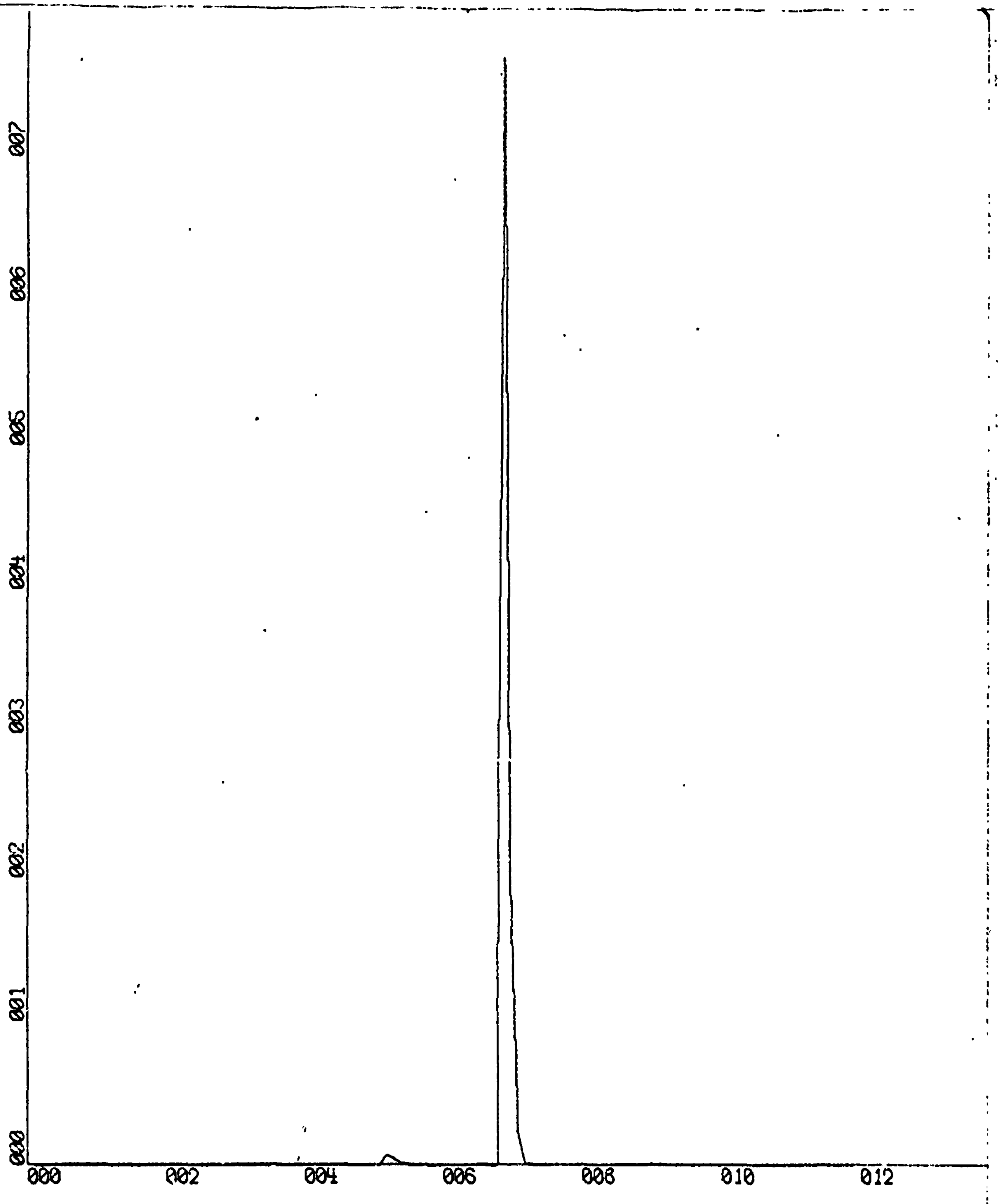

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Y-SCALE = 1.00E-01 UNITS/INCH.

E(A/C) =3.98E+00SPOTS = 8.00E+00 T = 2.00E

J VS QJ VECTOR N = 1.60E+01 A = 4.0

Q VECTOR CASE 3

```
 5     7
 1     1 QJ(  1 )=2.83568314E-05
 1     2 QJ(  2 )=2.15851385E-05
 1     3 QJ(  3 )=7.94343376E-06
 1     4 QJ(  4 )=1.91420746E-06
 1     5 QJ(  5 )=3.49125464E-07
 1     6 QJ(  6 )=5.37405166E-08
 1     7 QJ(  7 )=7.77939446E-09
 1     8 QJ(  8 )=1.18756605E-09
 1     9 QJ(  9 )=2.11283841E-10
 1    10 QJ( 10 )=5.18992660E-11
 1    11 QJ( 11 )=1.08528620E-11
 1    12 QJ( 12 )=1.87739309E-12
 1    13 QJ( 13 )=2.57762883E-13
 1    14 QJ( 14 )=2.63176291E-14
 1    15 QJ( 15 )=1.77499822E-15
 1    16 QJ( 16 )=5.92695364E-17
 1    17 QJ( 17 )=              0
 2     1 QJ( 18 )=4.81711691E-04
 2     2 QJ( 19 )=3.26173533E-04
 2     3 QJ( 20 )=1.04905695E-04
 2     4 QJ( 21 )=2.16142330E-05
 2     5 QJ( 22 )=3.28306882E-06
 2     6 QJ( 23 )=4.09033643E-07
 2     7 QJ( 24 )=4.68083562E-08
 2     8 QJ( 25 )=5.62586373E-09
 2     9 QJ( 26 )=8.10258169E-10
 2    10 QJ( 27 )=1.69317736E-10
 2    11 QJ( 28 )=2.92764116E-11
 2    12 QJ( 29 )=4.01871708E-12
 2    13 QJ( 30 )=4.10297009E-13
 2    14 QJ( 31 )=2.76746676E-14
 2    15 QJ( 32 )=9.24174352E-16
 2    16 QJ( 33 )=              0
 2    17 QJ( 34 )=              0
 3     1 QJ( 35 )=3.67589958E-03
 3     2 QJ( 36 )=2.22029313E-03
 3     3 QJ( 37 )=6.25702594E-04
 3     4 QJ( 38 )=1.10292585E-04
 3     5 QJ( 39 )=1.38900387E-05
 3     6 QJ( 40 )=1.38024736E-06
 3     7 QJ( 41 )=1.21057358E-07
 3     8 QJ( 42 )=1.09262020E-08
 3     9 QJ( 43 )=1.21362937E-09
 3    10 QJ( 44 )=2.09057619E-10
 3    11 QJ( 45 )=2.86055733E-11
 3    12 QJ( 46 )=2.91366712E-12
 3    13 QJ( 47 )=1.96282619E-13
 3    14 QJ( 48 )=6.55549374E-15
 3    15 QJ( 49 )=              0
 3    16 QJ( 50 )=              0
 3    17 QJ( 51 )=              0
 4     1 QJ( 52 )=1.66499369E-02
 4     2 QJ( 53 )=9.01023249E-03
 4     3 QJ( 54 )=2.23842146E-03
 4     4 QJ( 55 )=3.40006373E-04
 4     5 QJ( 56 )=3.57140995E-05
 4     6 QJ( 57 )=2.82593562E-06
 4     7 QJ( 58 )=1.85894467E-07
 4     8 QJ( 59 )=1.19355508E-08
 4     9 QJ( 60 )=9.51529557E-10
 4    10 QJ( 61 )=1.28538146E-10
 4    11 QJ( 62 )=1.28940846E-11
 4    12 QJ( 63 )=8.53320924E-13
 4    13 QJ( 64 )=2.79262670E-14
 4    14 QJ( 65 )=              0
 4    15 QJ( 66 )=              0
 4    16 QJ( 67 )=              0
 4    17 QJ( 68 )=              0
 5     1 QJ( 69 )=4.98253597E-02
 5     2 QJ( 70 )=2.42738832E-02
 5     3 QJ( 71 )=5.35895703E-03
 5     4 QJ( 72 )=7.10282973E-04
 5     5 QJ( 73 )=6.33588530E-05
 5     6 QJ( 74 )=4.08232868E-06
 5     7 QJ( 75 )=2.05024689E-07
 5     8 QJ( 76 )=9.26759069E-09
 5     9 QJ( 77 )=5.04461339E-10
 5    10 QJ( 78 )=5.13281492E-11
 5    11 QJ( 79 )=3.44456222E-12
 5    12 QJ( 80 )=1.14275058E-13
 5    13 QJ( 81 )=              0
 5    14 QJ( 82 )=              0
 5    15 QJ( 83 )=              0
 5    16 QJ( 84 )=              0
 5    17 QJ( 85 )=              0
 6     1 QJ( 86 )=1.04300023E-01
 6     2 QJ( 87 )=4.52895010E-02
 6     3 QJ( 88 )=8.79453771E-03
 6     4 QJ( 89 )=1.00690538E-03
 6     5 QJ( 90 )=7.55512291E-05
 6     6 QJ( 91 )=3.92418576E-06
 6     7 QJ( 92 )=1.47699319E-07
 6     8 QJ( 93 )=4.43630448E-09
 6     9 QJ( 94 )=1.43899935E-10
 6    10 QJ( 95 )=9.80396467E-12
 6    11 QJ( 96 )=3.30144045E-13
 6    12 QJ( 97 )=              0
 6    13 QJ( 98 )=              0
 6    14 QJ( 99 )=              0
 6    15 QJ(100 )=              0
 6    16 QJ(101 )=              0
 6    17 QJ(102 )=              0
```

```
  7    1 QJ(103 )=5.55639840E-01
  7    2 QJ(104 )=1.48858751E-01
  7    3 QJ(105 )=1.84227232E-02
  7    4 QJ(106 )=1.37794578E-03
  7    5 QJ(107 )=6.83380035E-05
  7    6 QJ(108 )=2.32262953E-06
  7    7 QJ(109 )=5.43332378E-08
  7    8 QJ(110 )=8.80653967E-10
  7    9 QJ(111 )=1.18536160E-11
  7   10 QJ(112 )=3.82356527E-13
  7   11 QJ(113 )=                0
  7   12 QJ(114 )=                0
  7   13 QJ(115 )=                0
  7   14 QJ(116 )=                0
  7   15 QJ(117 )=                0
  7   16 QJ(118 )=                0
  7   17 QJ(119 )=                0
QSUM=1.000000000E 00
EAN A/C FLYING =5.5636E 00

GRAPH TITLED
     E(A/C) =5.56E+00SPOTS = 8.00E+00     T = 3.00E+00
     J VS QJ VECTOR       N = 1.50E+01     A = 6.00E+00
HAS BEEN PLOTTED.
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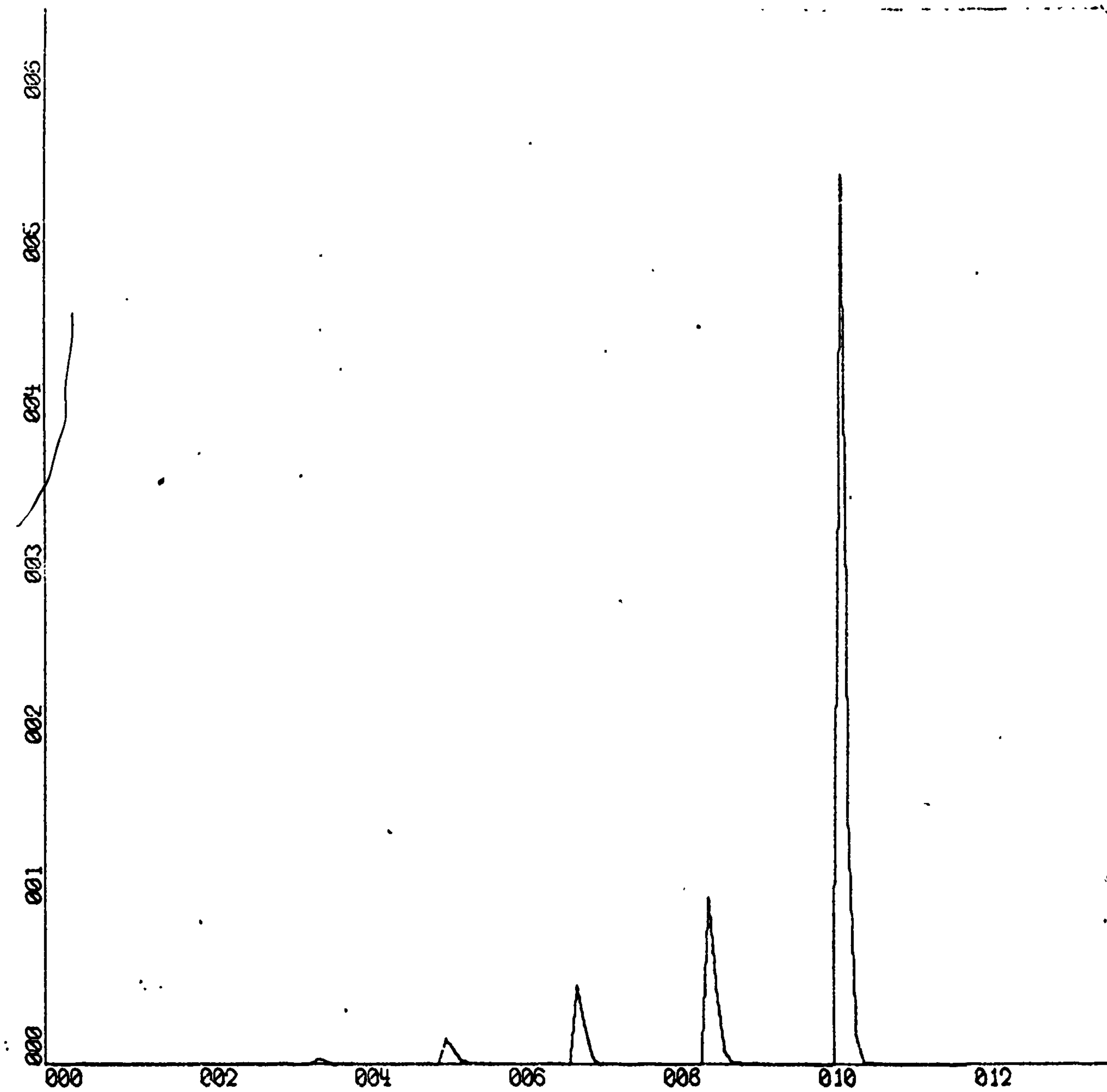

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E-01 UNITS/INCH.

E(A/C) =5.56E+00SPOTS = 8.00E+00 T = 3.00E

J VS QJ VECTOR N = 1.50E+01 A = 6.0

Q VECTOR CASE 4

```
 7     5
1   1 QJ(  1 )=4.37066460E-04
1   2 QJ(  2 )=2.66337542E-04
1   3 QJ(  3 )=7.61707863E-05
1   4 QJ(  4 )=1.37453763E-05
1   5 QJ(  5 )=1.79617995E-06
1   6 QJ(  6 )=1.89112432E-07
1   7 QJ(  7 )=1.81098891E-08
1   8 QJ(  8 )=1.84326720E-09
1   9 QJ(  9 )=2.33815659E-10
1  10 QJ( 10 )=4.51123736E-11
1  11 QJ( 11 )=7.18762832E-12
1  12 QJ( 12 )=9.07642678E-13
1  13 QJ( 13 )=8.51451858E-14
1  14 QJ( 14 )=5.27326607E-15
1  15 QJ( 15 )=1.61678538E-16
1  16 QJ( 16 )=               0
1  17 QJ( 17 )=               0
2   1 QJ( 18 )=5.18126447E-03
2   2 QJ( 19 )=2.86433504E-03
2   3 QJ( 20 )=7.33497609E-04
2   4 QJ( 21 )=1.16340651E-04
2   5 QJ( 22 )=1.30125431E-05
2   6 QJ( 23 )=1.13029797E-06
2   7 QJ( 24 )=8.54466567E-08
2   8 QJ( 25 )=6.65517506E-09
2   9 QJ( 26 )=6.57969504E-10
2  10 QJ( 27 )=1.05656138E-10
2  11 QJ( 28 )=1.34342964E-11
2  12 QJ( 29 )=1.26766173E-12
2  13 QJ( 30 )=7.88829950E-14
2  14 QJ( 31 )=2.42720088E-15
2  15 QJ( 32 )=               0
2  16 QJ( 33 )=               0
2  17 QJ( 34 )=               0
3   1 QJ( 35 )=2.69210521E-02
3   2 QJ( 36 )=1.34302284E-02
3   3 QJ( 37 )=3.06604116E-03
3   4 QJ( 38 )=4.26067113E-04
3   5 QJ( 39 )=4.06841633E-05
3   6 QJ( 40 )=2.90134684E-06
3   7 QJ( 41 )=1.70443704E-07
3   8 QJ( 42 )=9.76651595E-09
3   9 QJ( 43 )=7.11954817E-10
3  10 QJ( 44 )=9.16366982E-11
3  11 QJ( 45 )=8.74738102E-12
3  12 QJ( 46 )=5.50237415E-13
3  13 QJ( 47 )=1.70994033E-14
3  14 QJ( 48 )=               0
3  15 QJ( 49 )=               0
3  16 QJ( 50 )=               0
3  17 QJ( 51 )=               0
4   1 QJ( 52 )=8.03893389E-02
4   2 QJ( 53 )=3.58842827E-02
4   3 QJ( 54 )=7.24458728E-03
4   4 QJ( 55 )=8.75756513E-04
4   5 QJ( 56 )=7.09643835E-05
4   6 QJ( 57 )=4.12757545E-06
4   7 QJ( 58 )=1.85478755E-07
4   8 QJ( 59 )=7.45697706E-09
4   9 QJ( 60 )=3.67973791E-10
4  10 QJ( 61 )=3.56621692E-11
4  11 QJ( 62 )=2.27683580E-12
4  12 QJ( 63 )=7.17866372E-14
4  13 QJ( 64 )=               0
4  14 QJ( 65 )=               0
4  15 QJ( 66 )=               0
4  16 QJ( 67 )=               0
4  17 QJ( 68 )=               0
5   1 QJ( 69 )=6.25520845E-01
5   2 QJ( 70 )=1.72496943E-01
5   3 QJ( 71 )=2.20943200E-02
5   4 QJ( 72 )=1.73104557E-03
5   5 QJ( 73 )=9.18785067E-05
5   6 QJ( 74 )=3.46552930E-06
5   7 QJ( 75 )=9.61462191E-08
5   8 QJ( 76 )=2.13537750E-09
5   9 QJ( 77 )=5.31168734E-11
5  10 QJ( 78 )=3.22108252E-12
5  11 QJ( 79 )=9.66869152E-14
5  12 QJ( 80 )=               0
5  13 QJ( 81 )=               0
5  14 QJ( 82 )=               0
5  15 QJ( 83 )=               0
5  16 QJ( 84 )=               0
5  17 QJ( 85 )=               0
6   1 QJ( 86 )=               0
6   2 QJ( 87 )=               0
6   3 QJ( 88 )=               0
6   4 QJ( 89 )=               0
6   5 QJ( 90 )=               0
6   6 QJ( 91 )=               0
6   7 QJ( 92 )=               0
6   8 QJ( 93 )=               0
6   9 QJ( 94 )=               0
6  10 QJ( 95 )=               0
6  11 QJ( 96 )=               0
6  12 QJ( 97 )=               0
6  13 QJ( 98 )=               0
6  14 QJ( 99 )=               0
6  15 QJ(100 )=               0
6  16 QJ(101 )=               0
6  17 QJ(102 )=               0
```

```
7    1 QJ(103 )=              0
7    2 QJ(104 )=              0
7    3 QJ(105 )=              0
7    4 QJ(106 )=              0
7    5 QJ(107 )=              0
7    6 QJ(108 )=              0
7    7 QJ(109 )=              0
7    8 QJ(110 )=              0
7    9 QJ(111 )=              0
7   10 QJ(112 )=              0
7   11 QJ(113 )=              0
7   12 QJ(114 )=              0
7   13 QJ(115 )=              0
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7   16 QJ(118 )=              0
7   17 QJ(119 )=              0
QSUM=1.000000000E 00
EAN A/C FLYING =3.7578E 00

GRAPH TITLED
    E(A/C) =3.76E+00SPOTS = 8.00E+00     T = 4.00E+00
    J VS QJ VECTOR      N = 1.40E+01     A = 4.00E+00
HAS BEEN PLOTTED.
```

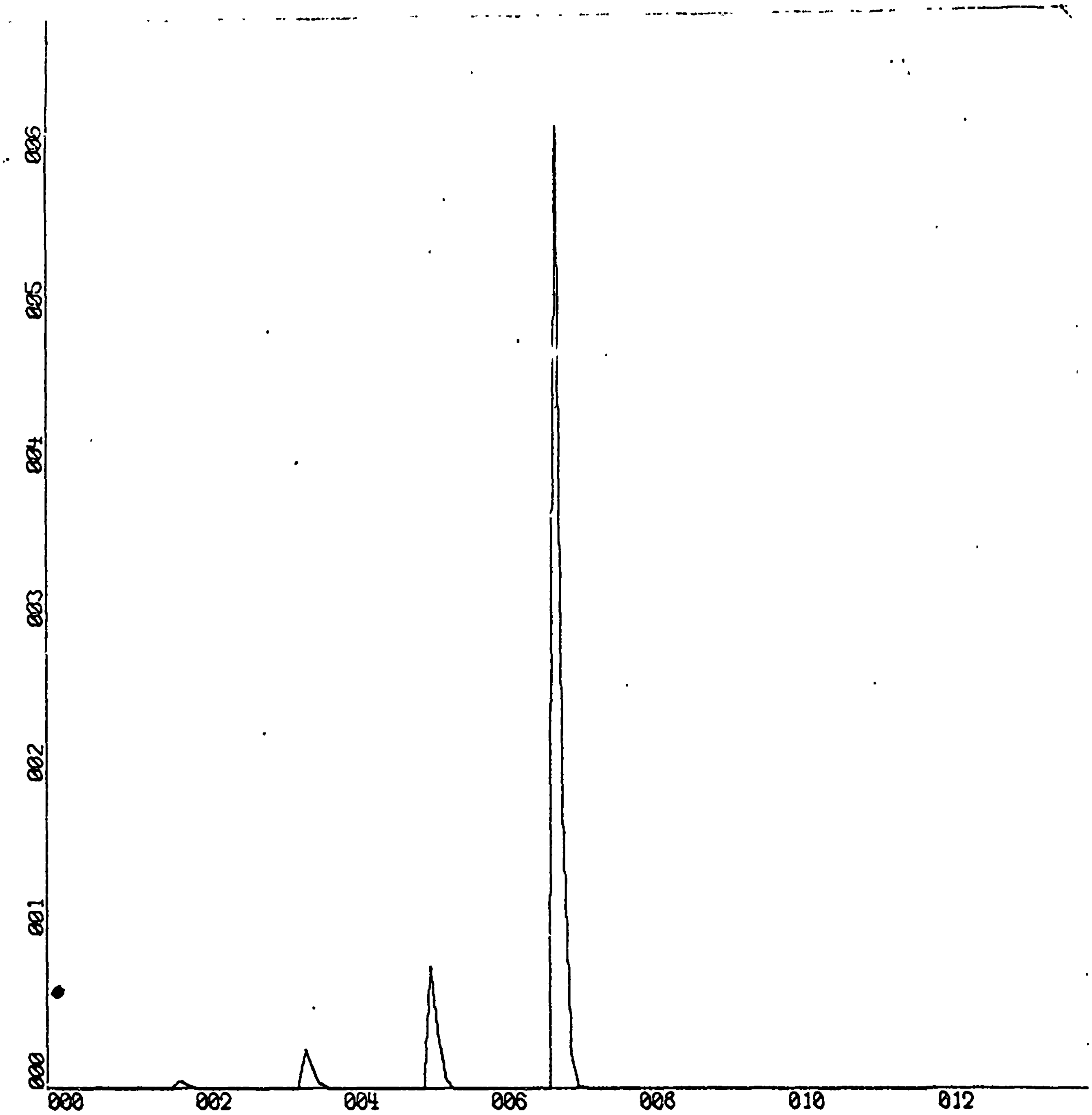

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Y-SCALE = 1.00E-01 UNITS/INCH.

E(A/C) =3.76E+00SPOTS = 8.00E+00 T = 4.00E-
J VS QJ VECTOR N = 1.40E+01 A = 4.00

Q VECTOR CASE 5

```
 5     7
 1     1 QJ(  1 )=7.15520995E-05
 1     2 QJ(  2 )=4.87189674E-05
 1     3 QJ(  3 )=1.57753676E-05
 1     4 QJ(  4 )=3.27727099E-06
 1     5 QJ(  5 )=5.02868635E-07
 1     6 QJ(  6 )=6.34199800E-08
 1     7 QJ(  7 )=7.35880233E-09
 1     8 QJ(  8 )=8.96925196E-10
 1     9 QJ(  9 )=1.30710966E-10
 1    10 QJ( 10 )=2.75401174E-11
 1    11 QJ( 11 )=4.80498022E-12
 1    12 QJ( 12 )=6.66122816E-13
 1    13 QJ( 13 )=6.87539584E-14
 1    14 QJ( 14 )=4.69377136E-15
 1    15 QJ( 15 )=1.58863224E-16
 1    16 QJ( 16 )=             0
 1    17 QJ( 17 )=             0
 2     1 QJ( 18 )=1.08771615E-03
 2     2 QJ( 19 )=6.60751603E-04
 2     3 QJ( 20 )=1.87506066E-04
 2     4 QJ( 21 )=3.33393976E-05
 2     5 QJ( 22 )=4.24508484E-06
 2     6 QJ( 23 )=4.27752034E-07
 2     7 QJ( 24 )=3.81603116E-08
 2     8 QJ( 25 )=3.50828896E-09
 2     9 QJ( 26 )=3.95927530E-10
 2    10 QJ( 27 )=6.89194802E-11
 2    11 QJ( 28 )=9.53886847E-12
 2    12 QJ( 29 )=9.83824310E-13
 2    13 QJ( 30 )=6.71883385E-14
 2    14 QJ( 31 )=2.27773386E-15
 2    15 QJ( 32 )=             0
 2    16 QJ( 33 )=             0
 2    17 QJ( 34 )=             0
 3     1 QJ( 35 )=7.37121582E-03
 3     2 QJ( 36 )=4.01163961E-03
 3     3 QJ( 37 )=1.00338550E-03
 3     4 QJ( 38 )=1.53687580E-04
 3     5 QJ( 39 )=1.63163199E-05
 3     6 QJ( 40 )=1.30932636E-06
 3     7 QJ( 41 )=8.77478713E-08
 3     8 QJ( 42 )=5.76387628E-09
 3     9 QJ( 43 )=4.69636023E-10
 3    10 QJ( 44 )=6.43465918E-11
 3    11 QJ( 45 )=6.56029037E-12
 3    12 QJ( 46 )=4.42292681E-13
 3    13 QJ( 47 )=1.47863511E-14
 3    14 QJ( 48 )=             0
 3    15 QJ( 49 )=             0
 3    16 QJ( 50 )=             0
 3    17 QJ( 51 )=             0
 4     1 QJ( 52 )=2.93940831E-02
 4     2 QJ( 53 )=1.43984862E-02
 4     3 QJ( 54 )=3.19845745E-03
 4     4 QJ( 55 )=4.26970762E-04
 4     5 QJ( 56 )=3.84133994E-05
 4     6 QJ( 57 )=2.50141306E-06
 4     7 QJ( 58 )=1.27347959E-07
 4     8 QJ( 59 )=5.85531451E-09
 4     9 QJ( 60 )=3.24180085E-10
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 4    11 QJ( 62 )=2.26209155E-12
 4    12 QJ( 63 )=7.60530690E-14
 4    13 QJ( 64 )=             0
 4    14 QJ( 65 )=             0
 4    15 QJ( 66 )=             0
 4    16 QJ( 67 )=             0
 4    17 QJ( 68 )=             0
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 5     2 QJ( 70 )=3.36780394E-02
 5     3 QJ( 71 )=6.58809654E-03
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 5     5 QJ( 73 )=5.76075719E-05
 5     6 QJ( 74 )=3.02551600E-06
 5     7 QJ( 75 )=1.15424789E-07
 5     8 QJ( 76 )=3.52590714E-09
 5     9 QJ( 77 )=1.16438952E-10
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 6     3 QJ( 88 )=9.10073278E-03
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 6     5 QJ( 90 )=5.62254445E-05
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 6     7 QJ( 92 )=6.58960130E-08
 6     8 QJ( 93 )=1.26580622E-09
 6     9 QJ( 94 )=1.99736156E-11
 6    10 QJ( 95 )=6.91079850E-13
 6    11 QJ( 96 )=             0
 6    12 QJ( 97 )=             0
 6    13 QJ( 98 )=             0
 6    14 QJ( 99 )=             0
 6    15 QJ(100 )=             0
 6    16 QJ(101 )=             0
 6    17 QJ(102 )=             0
```
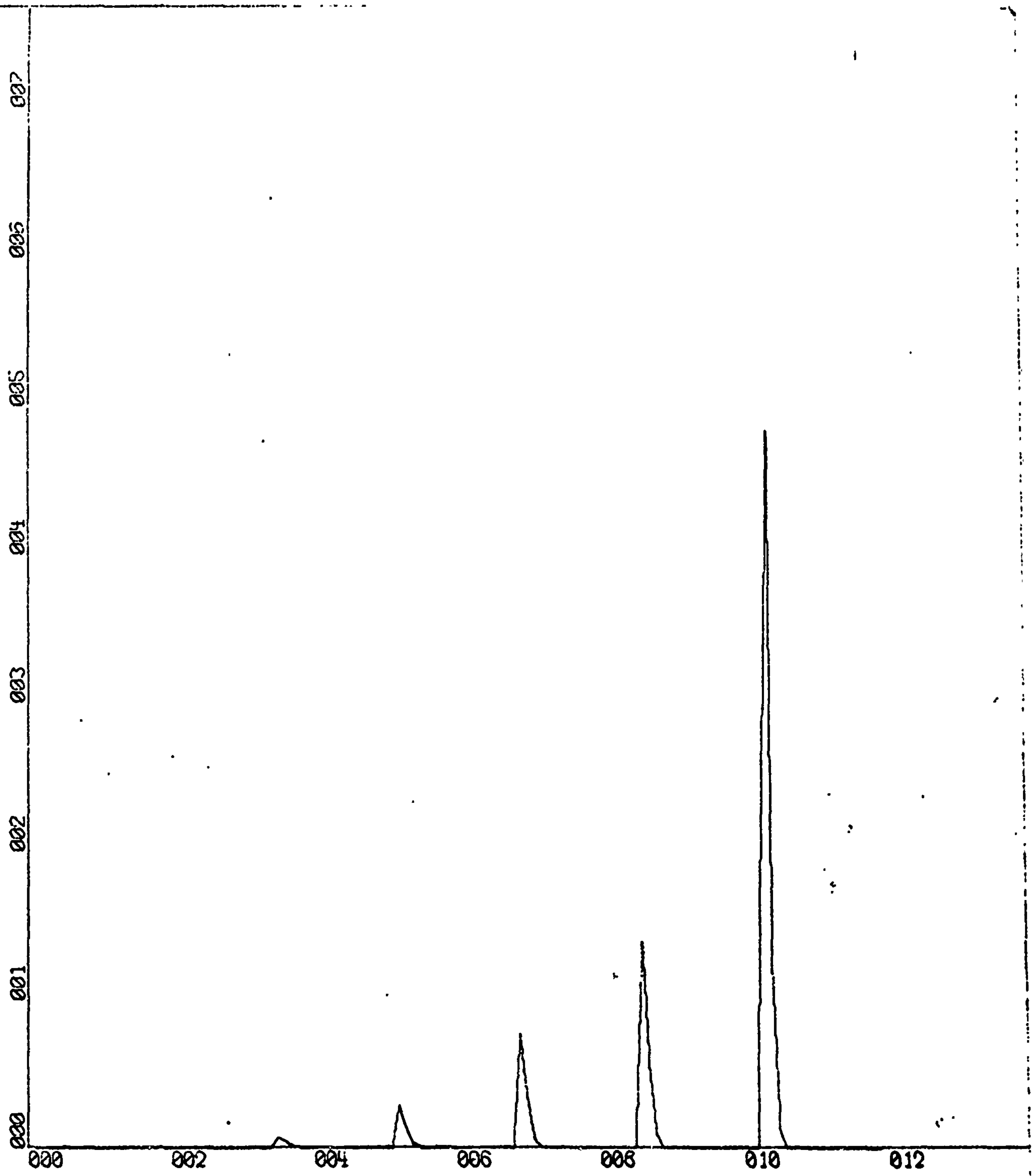

```
  7    1 QJ(103 )=4.84130317E-01
  7    2 QJ(104 )=1.18774563E-01
  7    3 QJ(105 )=1.31776649E-02
  7    4 QJ(106 )=8.59940961E-04
  7    5 QJ(107 )=3.59275288E-05
  7    6 QJ(108 )=9.79278393E-07
  7    7 QJ(109 )=1.69313196E-08
  7    8 QJ(110 )=1.69139063E-10
  7    9 QJ(111 )=7.45214538E-13
  7   10 QJ(112 )=                0
  7   11 QJ(113 )=.               0
  7   12 QJ(114 )=                0
  7   13 QJ(115 )=                0
  7   14 QJ(116 )=                0
  7   15 QJ(117 )=                0
  7   16 QJ(118 )=                0
  7   17 QJ(119 )=                0
QSUM=1.000000000E 00
EAN A/C FLYING =5.3577E 00

GRAPH TITLED
     E(A/C) =5.36E+00SPOTS = 8.00E+00     T = 5.00E+00
     J VS QJ VECTOR       N = 1.40E+01     A = 6.00E+00
HAS BEEN PLOTTED.
```



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E(A/C) =5.36E+00SPOTS = 8.00E+00 T = 5.00E·

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4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Master's Thesis

5. AUTHOR(S) (Last name, first name, initial)

Lanman, George M., Lieutenant Commander, U.S. Navy

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13. ABSTRACT

It is the purpose of this paper to develop a useful mathematical model of ASW aircraft availability. The increasing emphasis of systems studies dictates the use of accurate and representative models of the ASW systems. At present, many studies are using essentially the same models developed during World War II. This paper is an attempt to make use of advanced theory in a more powerful and flexible model and to make the use of the model practical and verifiable.

The writer adapted the time homogeneous bivariate model as developed by F. C. Collins. This is a discrete time Markov process with a stochastic matrix of transition probabilities wherein the maintenance process is modeled as a pulsed input multiple server queue.

The model was programmed in FORTRAN 63 on the CDC 1604 and then modified to allow for variability in the input parameters. Other modifications include an increase in the size of the model to accommodate a 16-aircraft squadron, the largest ASW squadron at present, and an explicit form solution to the maintenance queueing equations.

DD FORM 1 JAN 64 1473

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